1.0
1.1
1.25
1.4
1.6
2.8
3.2
3.6
4.0
2.5
2.2
2.0
1.8
MICROCOPY RESOLUTION TEST CHART

MINNE

Math And Science Teaching project

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UNIT XII

ADDITION & SUBTRACTION

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1965

MATHEMATICS

FOR THE

ELEMENTARY SCHOOL

UNIT XII

Addition and Subtraction involving

Random numbers

Number line

Nomogram

Number charts

Story problems

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University of Minnesota

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used earlier versions of this material
and provided suggestions for
this revision

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Section A. Finding the sum using the single number line

A Word to the Teacher

The following unit introduces the child to various tools which can help him to visualize and understand the operation of finding a sum or a missing addend.

At this time let the child manipulate a single number line, two number lines, and the nomogram. Let him make and learn to read an addition chart.

Although it may seem that there is an overemphasis on mechanical manipulations, it is through these that the child gains a concrete feeling for addition on the number line.

Random number cards A, B, and C are provided. Card C has more difficult work. Use the cards as suggested in line with the worksheets. But, stop, and use them again, and again, when and how you wish, but according to the needs of your class. Be flexible. Do not hesitate to alter a worksheet by using card C instead of card A or B. Cover as many pairs of numbers as you wish or be selective.

The story problems may be done orally. If your class reads well try some sheets individually. Do not punish a child by refusing to help with the reading.

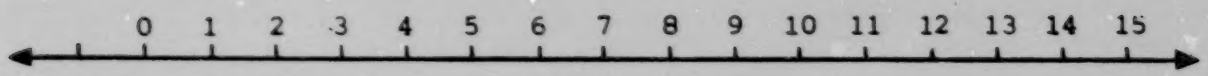
Be extremely cautious in selecting the proper scales for the 2-3 scale tools. Follow the directions carefully. Learning need not be tedious so games are introduced.

Read the complete portion of the unit so that you grasp its significance, then break it down to the bits your class can handle during one class period. Do not hurry the children. The physical handling of the tools requires time and practice.

Part 1. Oral work

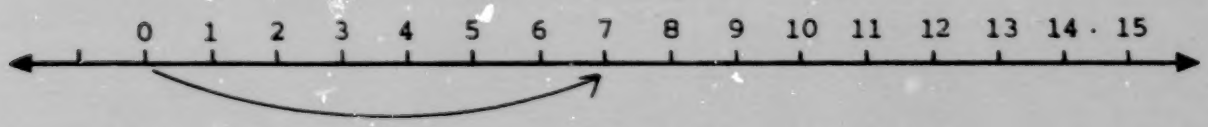
Draw three single number lines on chalkboard. Place the random number combinations as follows:

7,3



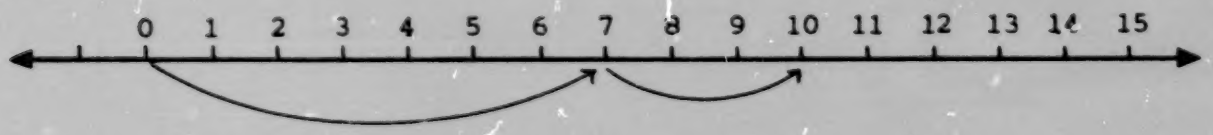
Today we are going to find the sum of two numbers using a single number line.
Who can show us the first number of the pair on the number line? Show how far it is from "0".
Like this:

7,3



Our next number is "3". This means a move of 3 units to the right of seven.
Like this:

7,3



Now who can tell us the sum of the numbers 7,3 as shown on the number line? (10).

provide each child with a number line worksheet for oral work.

8,7	9,4	4,0	4,7	6,4	2,5
4,9	5,6	2,8	7,2	0,6	3,9

Select one of the number combinations above. Ask the children to find the sum by referring to their number line. When it is apparent that most of the children have had ample time to find the sum, call upon any child to provide the answer and show it on the chalkboard diagram.

Repeat above for a third combination.

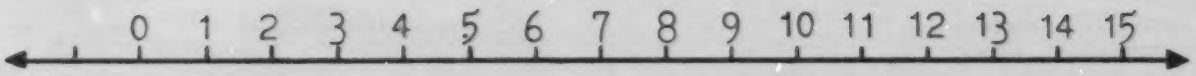
This procedure should be repeated for all the combinations. It should not be necessary to continue to use the chalkboard unless the children seem to be having difficulty. The children should be encouraged to practice the combinations whenever possible.

Worksheet 1 Name_____

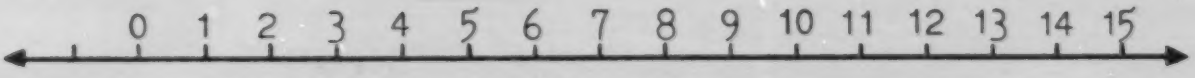
Let's do it together

Show how you would find the sum of two numbers on a number line.

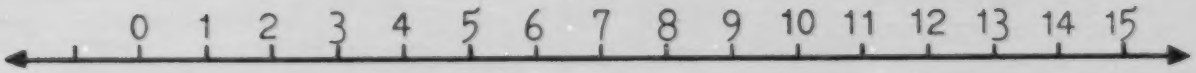
8.7



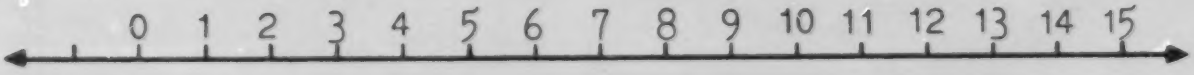
9.4



4.0



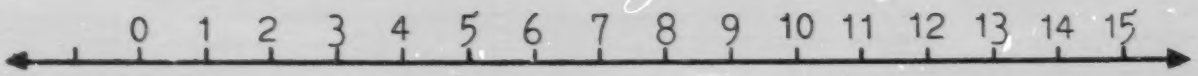
7.2



Worksheet 1 (con't)

Name_____

6.4



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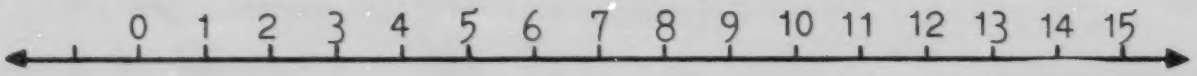

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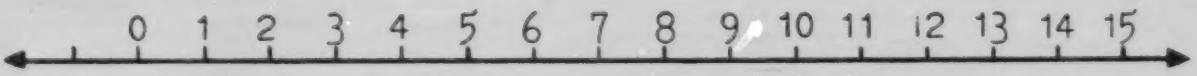
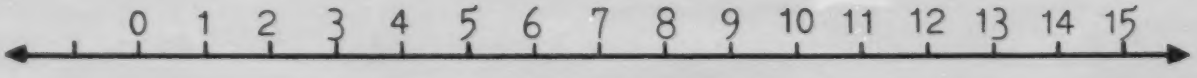


5.6



Part 2

Can you do it?

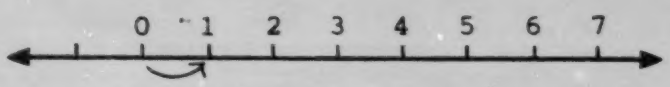
Provide each of the children with a worksheet. Be ready to use the Minnemast Slide Rule for demonstration purposes.

Today we are going to find the sum of two numbers using a single number line.

Let us look at our worksheets. Who can tell us the first number combination? (1,3).

Now who can show us the first number of the pair on our Minnemast number line? You will remember that we began at "0" and moved toward the right. Show this in pencil on your worksheet.

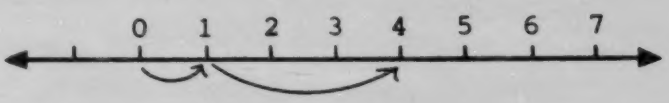
Like this



Who can show a move of three units from this point on the classroom number line?

Show this on your worksheet.

Like this:



We are now ready for someone to tell us the sum of 1,3 as shown on the number line (4).

Write the number sentence in the box below the number line (1 + 3 = 4).

Worksheet 2 Name_____

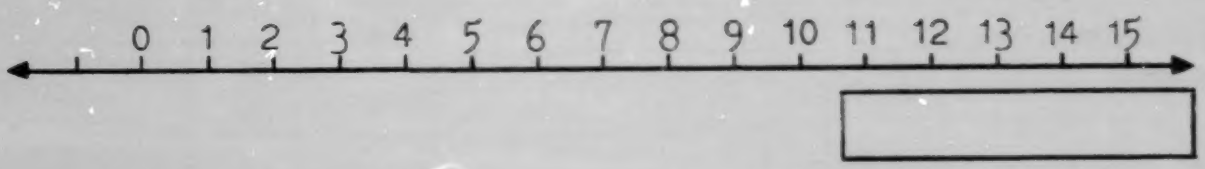
Can you do it?

Use the number line.

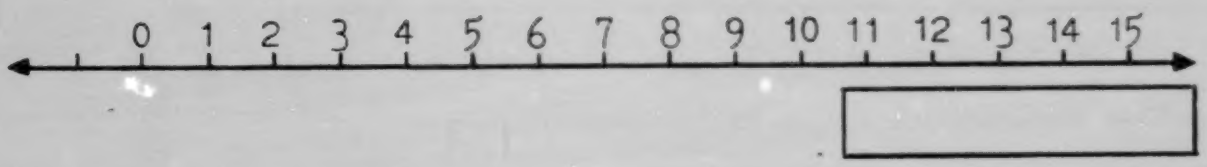
Find the sum of two numbers.

Write the number sentence.

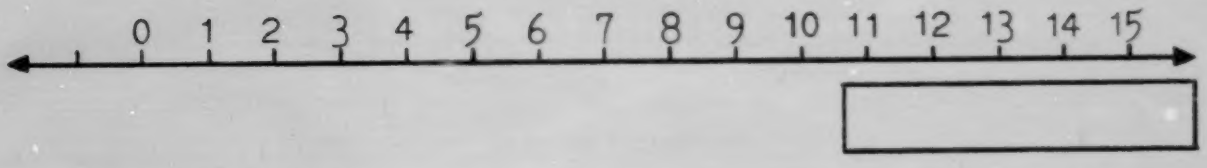
1,3



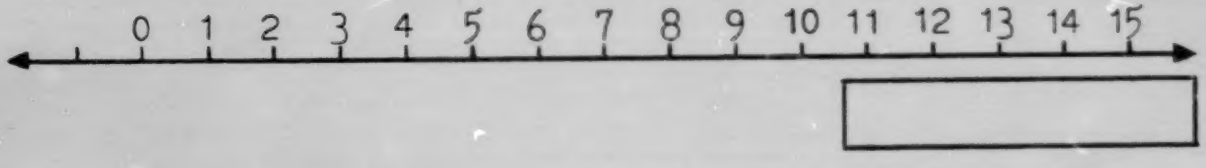
1,6



4,3



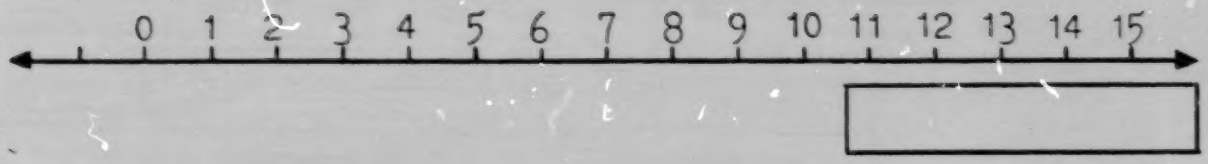
6,9



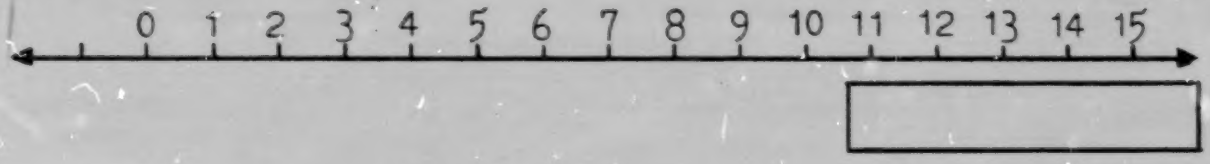
Worksheet 2 (con't)

Name_____

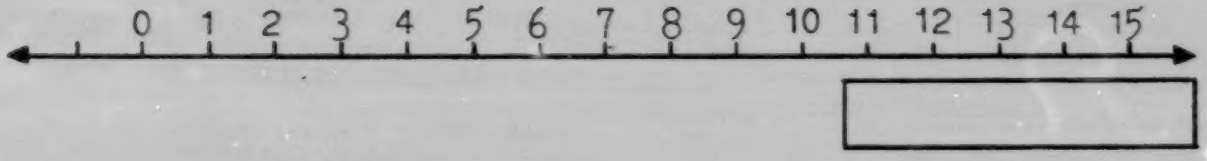
6.7



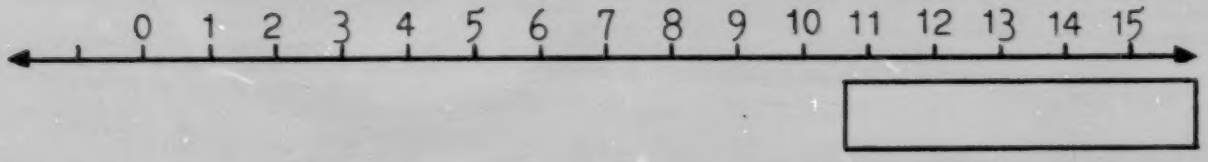
7.4



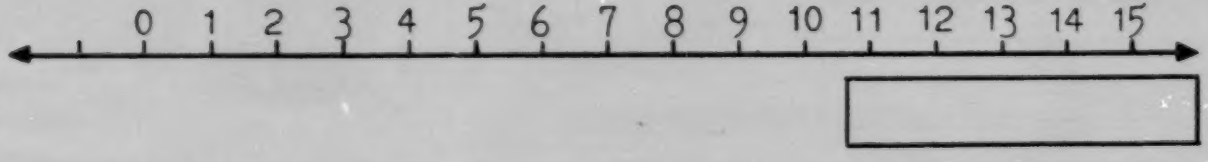
4.9



3.1



7.6



Part 3

A game to play: "Zip"

Provide each child with a plastic number line.

Divide the class into two teams. Designate one team as "X" and the other as "O". Select a caller for team "X" from team "O". Select a caller for team "O" from team "X".

Choose one child to be scorekeeper.

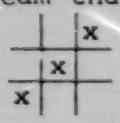
Draw on chalkboard

The teacher will then call each of these combinations alternating between the teams.

4,9	4,3	8,4	4,4	8,2	6,6	5,5	8,3	7,5
3,6	7,9	2,2	6,2	6,9	3,3	2,6	6,6	6,5

When a combination is called the children must find the sum on their plastic number lines. When team "X" is having its turn caller "O" will select anyone of his choosing to answer. (Because the caller represents the opposing team the participants will have to be attentive and alert so that when a child is selected at random he will be prepared with the answer.)

When the child who has been selected from team "X" answers correctly, he may go to the board and mark an "X" in one of the cross boxes. If he misses the team loses its turn to mark a box and team "O" now proceeds in the same manner. The team that wins has matched X's or O's in a straight line.



The score keeper then credits that team with a

point. The team which has the most points after all the number combinations have been called wins the game.

Part 4

Let's think

Provide each child with a worksheet. Draw a single number line diagram on chalkboard.

In our lesson today we will compare two pairs of number combinations.

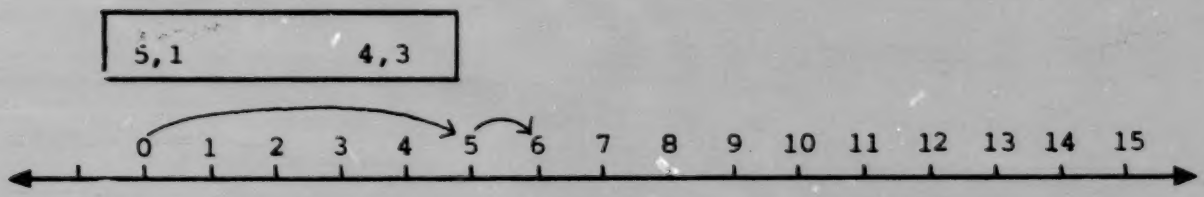
We will use the same single number line to find which of the pairs has the greater sum.

Let us look at the first pair of combinations (5,1 4,3).

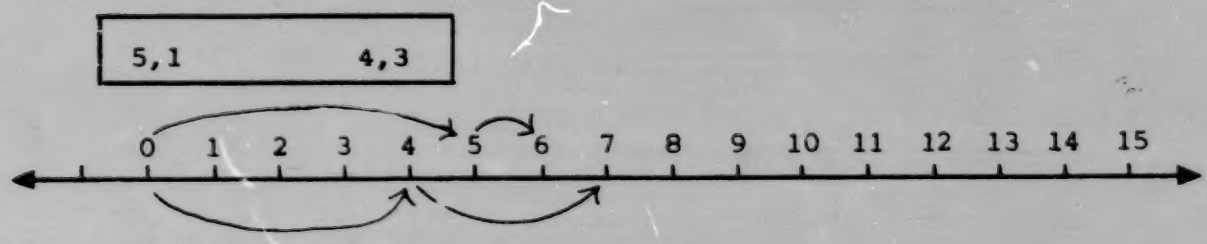
Which do you think has the greater sum? (4,3) Let us circle it.

Who can draw the lines which show the movement of units along the number line for the first combination (5,1)?

We can do this on the top of the line.
Like this:



In order to compare the two pairs of numbers we will show the second combination below the same line (4,3).
Like this:



We can see how the numbers compare by noting the lengths of the units marked off.

Was your estimate correct?

In the box below the line write the mathematical sentence which states that the sum of one is greater than the sum of the other.

Do it like this:

$$4 + 3 > 5 + 1$$

On your worksheets compare the other combinations in the same way.

It is anticipated that the children will discover the various ways to estimate when they see the relationship between the numbers they have compared.

Worksheet 3 Name_____

Which sum is greater?

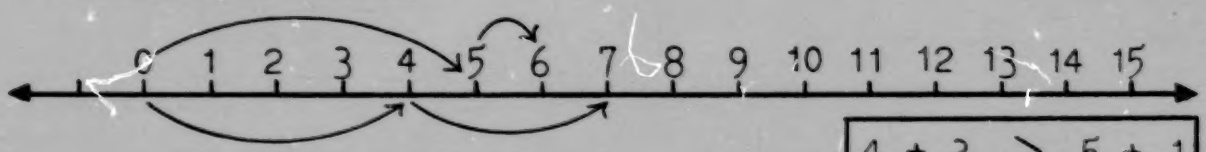
Which pair of numbers will have the greater sum?

Circle it.

Now see if you are correct.

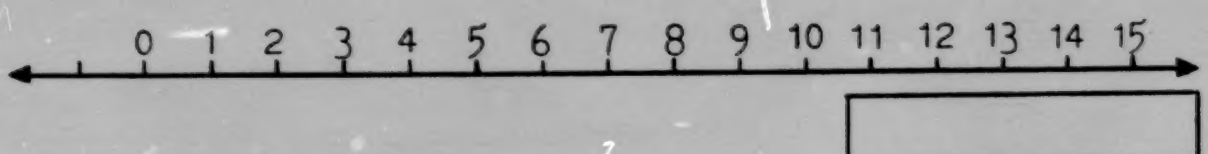
Use the number line like this:

5,1	4,3

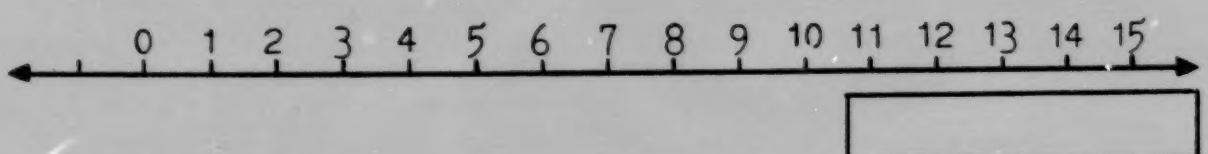


4 + 3 > 5 + 1

8,7	7,7



7,5	3,8

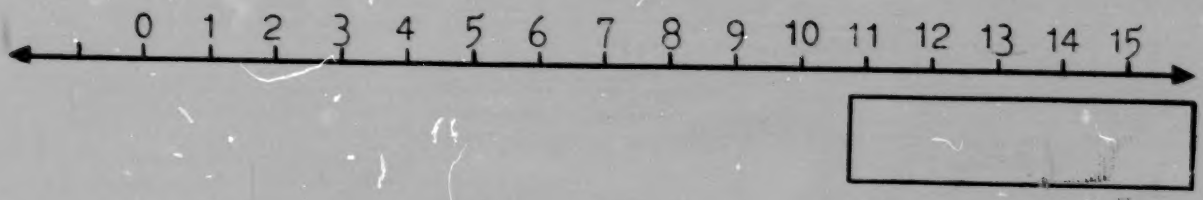


Worksheet 4

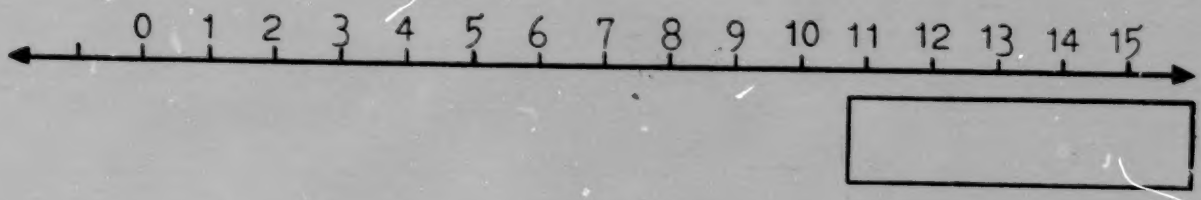
Name_____

Which sum is greater?(con't)

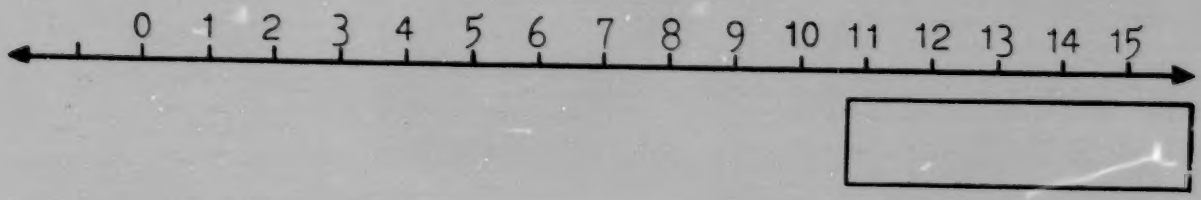
8,4 4,8



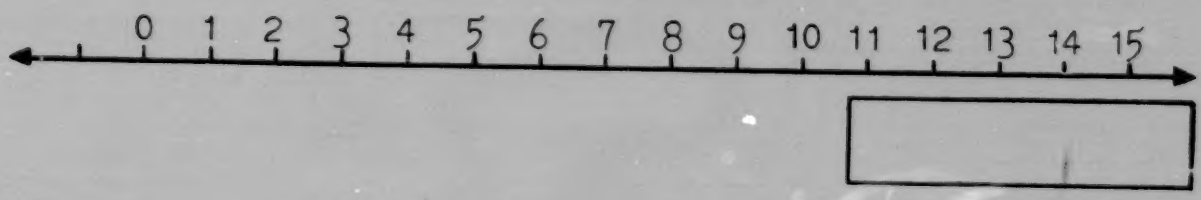
1,8 3,7



2,0 ?,1



7,0 5,1



Part 5

Fun with your buddy

Provide each child with Worksheet 5 . Assign half of the class as players number "one" and the other half as players number "two".

	1	2	3	4	5	6	7	8
A	9,8	7,5	6,2	6,3	6,0	6,4	5,1	6,1
B	7,1	3,2	5,5	5,2	1,7	1,3	0,1	5,7

Every child whose number is "one" should have a number "two" buddy.

"Two" players are to find the sums of number combinations in columns 2,4,6,8 by using the number line.

"One" players are to find the sums of number combinations in columns 1,3,5,7 by using the number line.

The object of the game is to see which of the two buddies can complete line A with the most speed and accuracy.

The buddy who has correctly finished the combination first scores one point.

The same procedure is carried out for line B.

Each child keeps his own score.

If the buddy who has finished first does not have all the answers correct, the other buddy earns the point.

This is determined when the entire class participates in the checking of answers.

Name_____

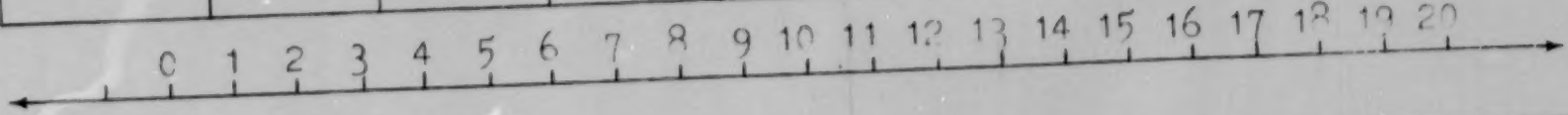
Worksheet 5

Buddy Game

Buddy number one finds the sums of combinations in odd numbered columns. Do row A and then row B. Use the number line to find your answer.

Buddy number two finds the sums of combinations in even numbered columns. Do row A and then row B. Use the number line to find your answer.

	1	2	3	4	5	6	7	8
A	9,8	7,5	6,2	6,3	6,0	6,4	5,1	6,1
B	7,1	3,2	5,5	5,2	1,7	1,3	0,1	5,7



	1	2	3	4	5	6	7	8
A								
B								

score A _____ + score B = total _____

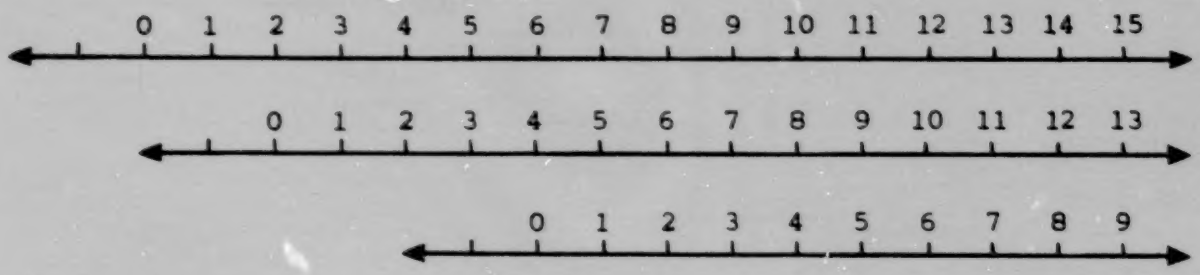
XII-15

Background for the Teacher

Understanding the property of associativity of addition enables a student to manipulate more easily any given problem which involves addition. In order to grasp this idea it is first necessary to re-examine what is meant by addition. We have presented addition by means of the number line. Let us recall how we proceed in the following problem:

$$\begin{array}{r} 2 \\ 4 \\ +7 \\ \hline \end{array}$$

Since there are three addends we will need three number lines to conveniently display the problem.



On the top line we marked the point corresponding to 2 then moved the next number line to align the 0 on that line with the 2 on the line above.

Finally we moved the bottom line laterally so that the 0 on that line is aligned with the 4 on the middle line and find the 7 on the third line. On the top line at a point aligned with the 7 (on the third line) is the sum.

There is an order to our work which corresponds to adding down in the original problem in like manner we might start in the reverse order and add up instead.

Two things are now observed (i) addition can be performed only on two numbers at a time and (ii) the sum is the same in both of the above suggested approaches.

To say it another way we can add 2 and 4 and obtain 6, 6 and 7 to get 13. We can add 4 and 7 to obtain 11, 2 and 11 to obtain 13. We have, of course, not proved that this is the case for all possible trios of numbers but our example makes it seem plausible.

In this particular problem it is possible to check the sum both ways. It would be inefficient to do every problem that way. At some point in the process of discovering sums

on the number line the student should be encouraged to generalize and grasp in an intuitive way the associative property of addition. Another example of how the student might use this new understanding is in filling in the entries in the addition table below:

	310	311	312	314
310				
311			623	
312				
313				

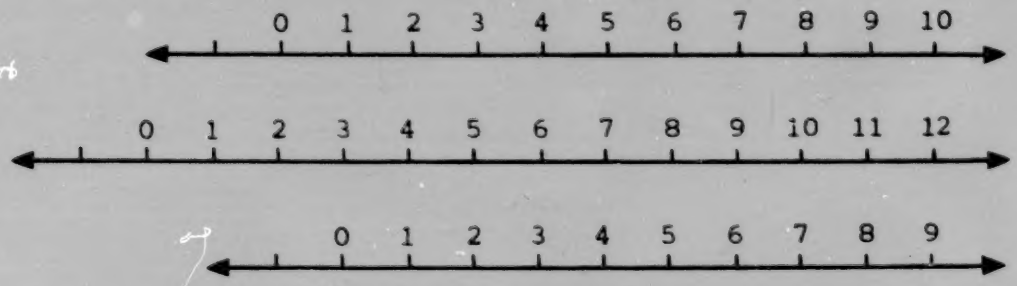
One sum has been supplied (311 + 312 = 623) and the student is asked to supply other entries using only very elementary knowledge about addition. To obtain 311 + 313 we re-write the problem as 311 + 312 + 1 = 623 + 1 = 624.

This also allows us to find that 311 + 311 + 1 + 1 = 623 + 1 = 622 + 1 + 1 or that 311 + 311 = 622.

From this we have 311 + 310 + 1 = 622 = 621 + 1 or 311 + 310 = 621.

For the remaining entries it is convenient to use another property of addition, commutativity. Having been given 311 + 312 = 623 can we conclude that 312 + 311 = 623? Can we also assert that 310 + 311 = 621 from the above statement that 311 + 310 = 621?

Returning again to the number line we find that another property seems likely and we illustrate it below:



To obtain 2 + 3 using the number lines above we find the point corresponding to 2 on the middle line and then slide the top line until the point at 0 on the top line is aligned with the point at 2 on the middle line.

The sum required is then found on the center line at the point which is aligned with the point at 3 on the center line and move the bottom line until the point at 0 on that line is aligned with the point at 3 on the center line.

Then the required sum is found on the center line at the point which is aligned with the point at 2 on the bottom line.

Having found that the sum is the same in each of these cases we are ready to ask whether this coincidence of sums depended in any way on the numbers 2 and 3.

By trying others we can determine that it seems very likely that for sums corresponding to any two points on the center line we find a similar result generalizing this we say that addition is commutative.

Now we can complete the above addition table by such applications of commutativity and associativity as is given in the following:

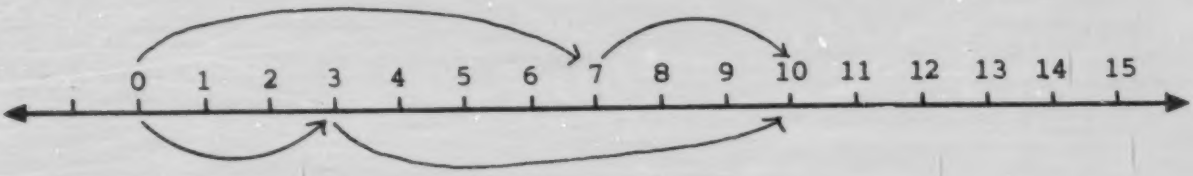
$$312 + 312 = 312 + 311 + 1 = 311 + 312 + 1 = 623 + 1 = 624$$

$$313 + 312 = 312 + 1 + 312 = 312 + 312 + 1 = 624 + 1 = 625$$

B. Discovering the commutative property using the number line

The commutative property states that, for example, 3 + 7 = 7 + 3 and the sum is "10" regardless of the order of the addends. This may seem very simple, but it takes well planned questioning and many experiences for the child to develop a correct understanding of this property.

Lead the child to see that this occurs on the number line.



In the following exercise lesser and greater numbers are presented to test the child's response.

Worksheet 6 Name_____

Commutativity I

Fill the _____to make a true statement.

What do you know that will help you?

A.

1) 15 + 6 = 6 + _____= 21

2) _____+ 8 = 8 + 12 = _____

3) 7 + 6 = _____+ 7 = _____

4) 9 + 3 = 3 + _____= _____

B.

1) 87 + 39 = _____+ 87

2) _____+ 48 = 48 + 59

3) 64 + 56 = 56 + _____

4) 38 + _____ = 92 + _____

Worksheet 7 Name_____

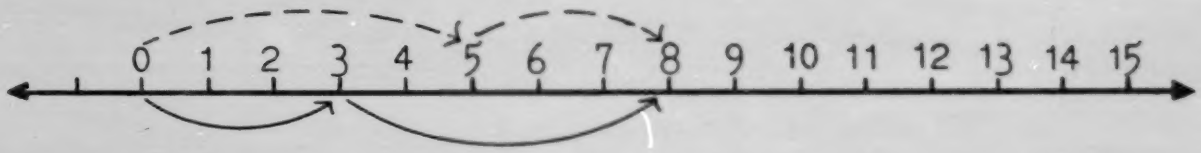
Commutativity II

Write the number fact in another order.

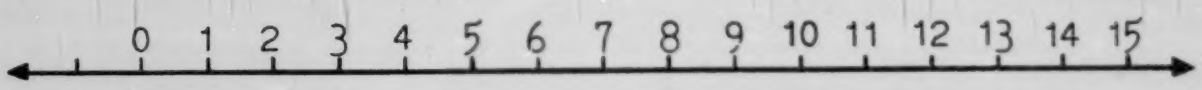
Use the number line to find the sum.

Do it like this:

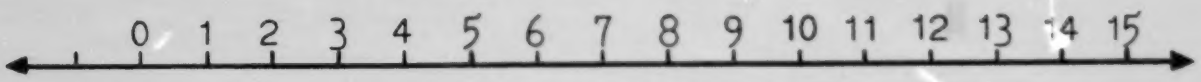
$3 + 5 = 5 + 3 = 8$



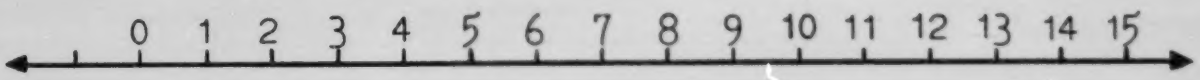
1. $6 + 4 =$ _____ = _____



2. $7 + 4 =$ _____ = _____



3. $5 + 4 =$ _____ = _____



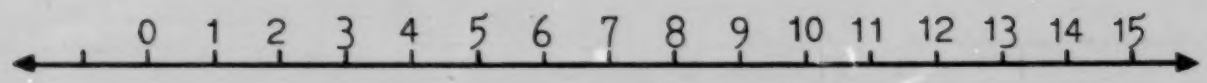
Worksheet 8 Name_____

Commutativity III

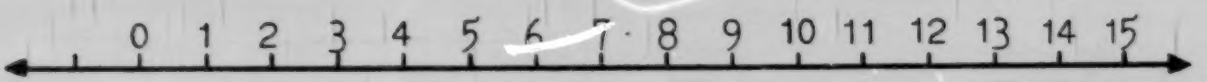
Write a numeral on the first line to make the statement true.

Find the sum on the number line and write the answer on the other line.

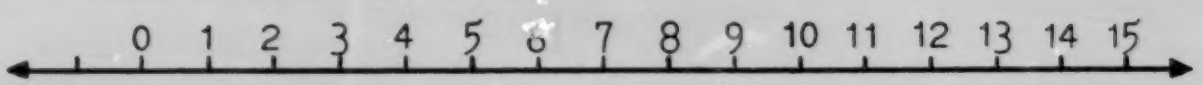
1. 2 + 6 = _____ + 2 = _____



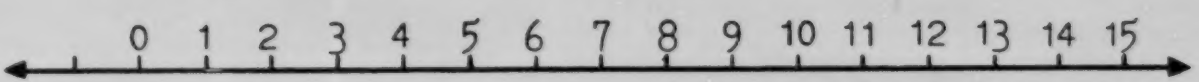
2. 4 + 3 = 3 + _____ = _____



3. 9 + 2 = _____ + 9 = _____



4. 6 + 7 = 7 + _____ = _____



Section C. Finding the missing addend using the single number line

Materials

1. Random number card A
2. Number lines to use
3. Rulers
4. Plastic Scale - one insert
5. Minnemast demonstration slide rule

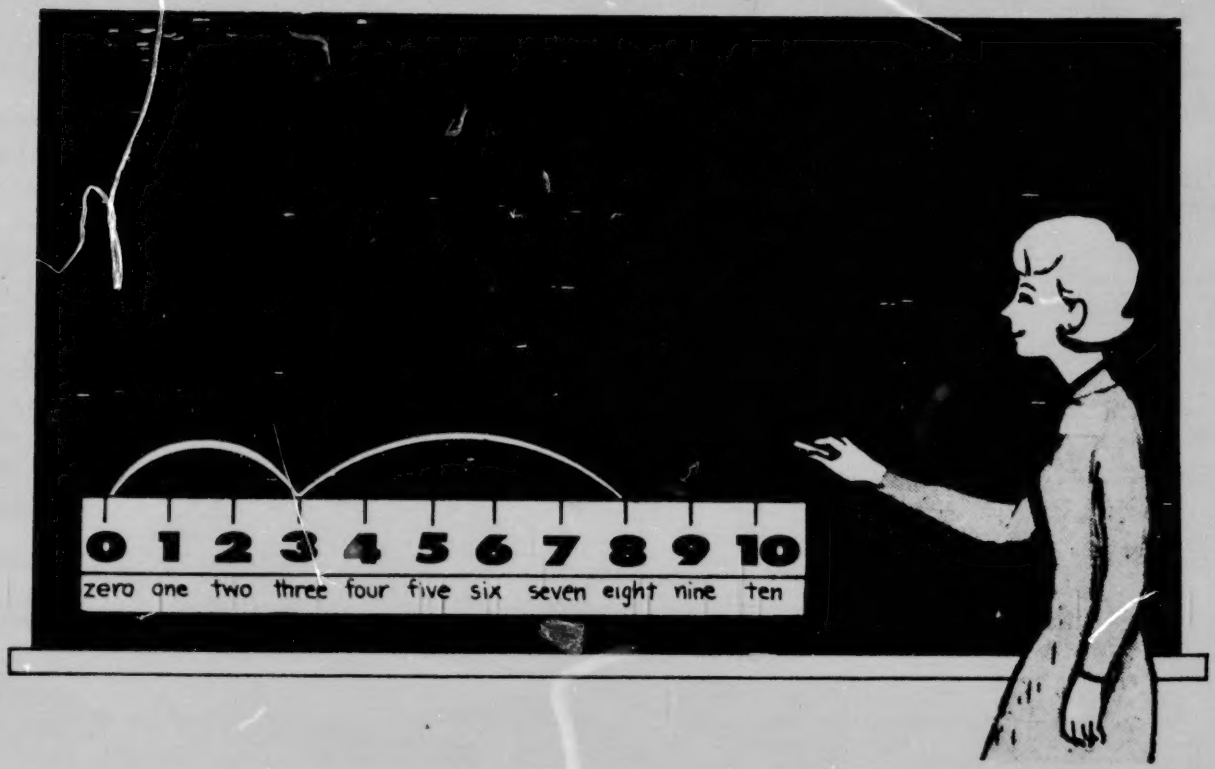
Introduction

5,8

To find the missing addend in examples such as 8 - 5 = ____, lead the class to think of a number sentence equivalent to the one given. Write these on the board. Having completed Unit XI Numeration, the children will recall that 5 + ___ = 8 satisfies this request.

In using the Random card numbers to find the missing addend ask the class to consider the greatest number as the sum. This leaves one known or given addend and they are to find the missing addend.

The Minnemast slide rule for the classroom is a very handy tool to use for demonstration purposes. Insert only one scale and stand it up on a chalk tray on a clean chalkboard to use right above it. Now show the moves above it like this:



Then lead the class:

1. to illustrate the move indicated by the known addend (This is "3" in the example.)
2. to proceed with a move to the point on the line labeled as the sum or "8". (Use a broken line to designate the unknown or missing addend.)

Discuss:

1. How many units did we move from 3 $\longrightarrow$ 8?
2. What is the missing addend?

Fill the blanks in the number sentences which are on the chalkboard.

$$8 - 3 = \underline{5}$$

$$5 + \underline{3} = 8$$

Ask the class to think of other number sentences which are closely related to these.

$$8 - 5 = 3$$

$$3 + 5 = 8$$

What relationship is there between 3 + 5 = 8 and 5 + 3 = 8. (Commutative Property)

Follow with more oral work. Use number pairs from Random card A such as:

8,2 7,3 3,9

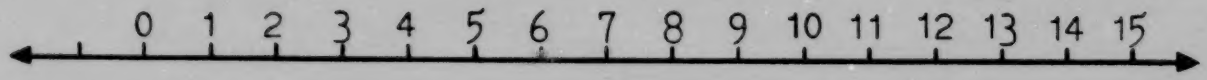
Worksheet 9 Name_____

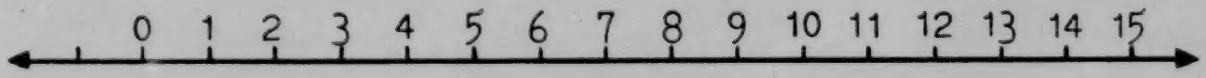
Random number card A

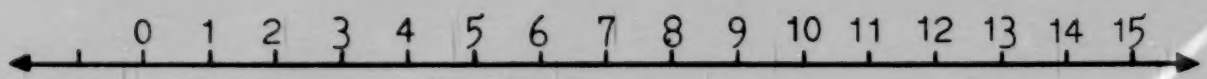
1)	1,1	1,6	4,3	6,3	1,8	7,5	1,6
2)	2,1	2,1	5,9	1,7	9,1	7,6	8,3
3)	1,0	4,3	8,4	4,4	8,2	6,6	5,5
4)	3,6	7,9	2,2	6,2	3,6	3,3	2,6
5)	7,3	9,4	4,0	4,7	7,3	1,2	0,3
6)	4,9	5,6	3,1	2,8	7,2	1,4	0,6
7)	6,4	2,0	8,4	8,2	3,7	4,1	7,0
8)	5,1	4,8	6,7	2,8	7,5	3,8	6,1
9)	9,9	7,5	6,2	6,3	6,0	6,4	5,1
10)	7,1	3,2	5,5	5,2	1,7	1,3	0,1

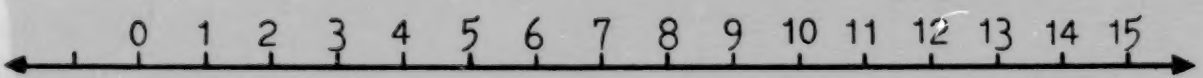
Worksheet 10 Name_____

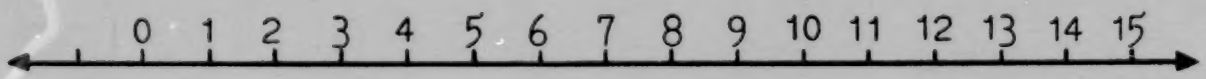
Number lines to use

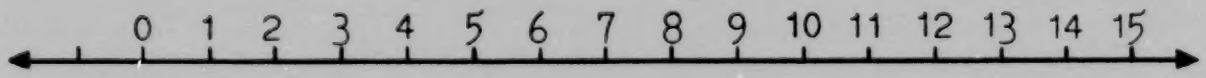












Worksheet 11 Name_____

Find the missing addend I

Use the number line on your ruler.

1. Make the move for the known addend.
2. Then make another move to the point which is the sum.
3. Fill in the _____.

6 - 3 = _____

5 + _____ = _____

6 + _____ = _____

2 + _____ = 6

4 + _____ = 8

9 - 1 = _____

5 + _____ = 7

4 + _____ = 8

2 + _____ = 5

3 + _____ = 8

2 + _____ = 7

9 - 2 = _____

9 - 3 = _____

7 + _____ = 9

2 + _____ = 8

5 - 2 = _____

Worksheet 12 Name_____

Find the missing addend II

Use Random number card A. Choose any pair of numbers. Use the single scale on your ruler or paper to help you. Fill the ☐ like this

1 + 4 = 5

A Game to Play

Use one scale on the Minnemast slide rule.

Place it in full view of every child.

Select 2 teams. Let them be seated across from each other.

A	B
A	B
A	B

The teacher writes a pair of numerals on the chalkboard from Random card A.

When the teacher says go, the first contestants run to the board to write the number sentence. Like this:

6,2

$2 + 4 = 6.$

Keep score for the teams. Speed and accuracy are required.

Worksheet 13 Name_____

Try your skill

Fold this sheet so the boxes are under line 4 of card A.

Look at the pair of numerals; the greater number is the sum.

Write the missing addend in the box below.

Fold your paper again for each new row.

4) ___ ___ ___ ___ ___ ___ ___

5) ___ ___ ___ ___ ___ ___ ___

6) ___ ___ ___ ___ ___ ___ ___

7) ___ ___ ___ ___ ___ ___ ___

8) ___ ___ ___ ___ ___ ___ ___

9) ___ ___ ___ ___ ___ ___ ___

10) ___ ___ ___ ___ ___ ___ ___

Section D. — Using two number lines to find the sum

Introduction

The motivation for the use of two number lines may be centered around the idea that tools are simple machines and machines make work easier for us. The analogy can then be extended so that children will realize that this mathematical device can make it easier for us to understand the concept of addition.

A variety of activities is provided to encourage the children to use the two number lines. The consistent use of this tool over a given period of time should reinforce their ability to see the one-to-one relationship between points and numbers which is inherent in the operation of addition on the number line.

The frequency with which they "find sums" will add to their "memory bank" of number facts.

Part 1

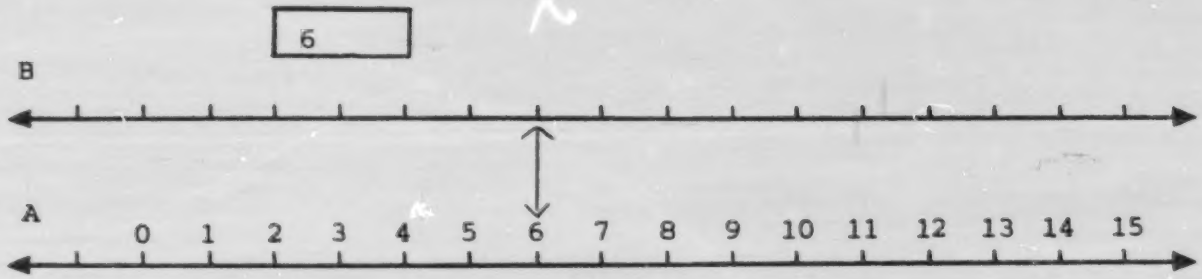
An easier way

Provide children with worksheets.
Draw two number lines on chalkboard. Number the points of the second line "A". Show the unnumbered points of the first line "B".

Let us see if by using the two number line we can better understand how we can find the sum.

Who can mark the first point on our classroom diagram? (6)
We will begin by using number line A.

Like this:



Now that you have found the point at 6 draw a line straight up so that it meets the matching point on line B. We can call that point "0" on line B.

Can someone suggest how we might find the sum of 6 + 5 by using line B?

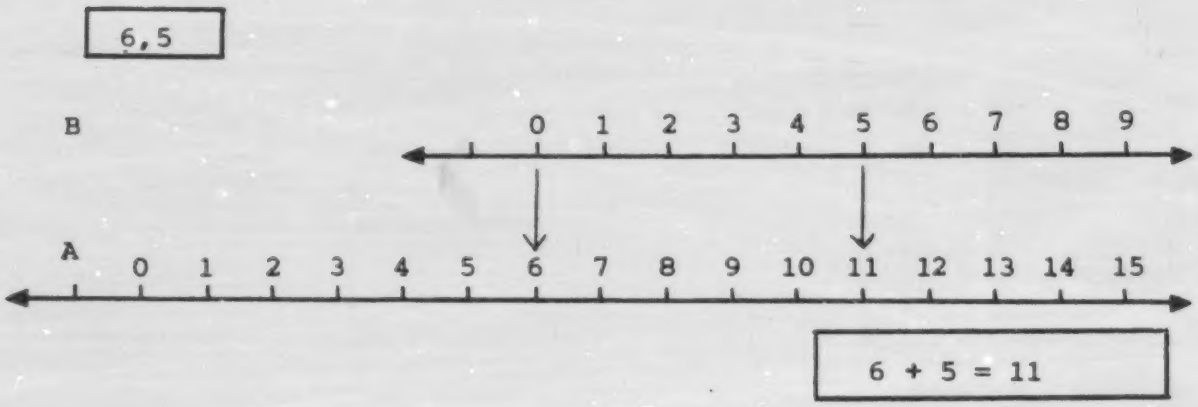
Yes, we can continue numbering from "0" on B to find the point at 5.

If we match the point at 5 with the point on line A we discover the point at 11. Draw an arrow from "5" on B to "11" on A.

With this useful tool we can understand why 6 plus 5 will give us the sum of 11.

Write the number sentence in the space below the lines.

Like this:



Worksheet 14 Name_____

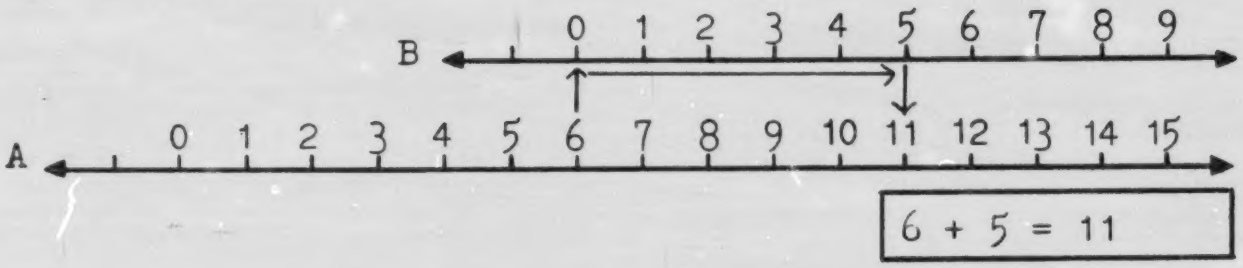
Find the sum

On line A find the point at the first number (6). Match this point with the point on B. We call this point "O".

Now finish numbering points of line B and find the second number.

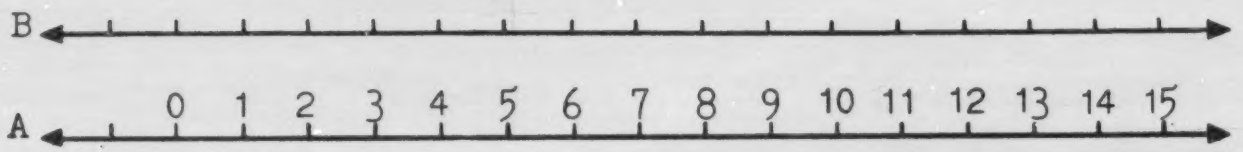
Write the number sentence in box below line.

6,5



6 + 5 = 11

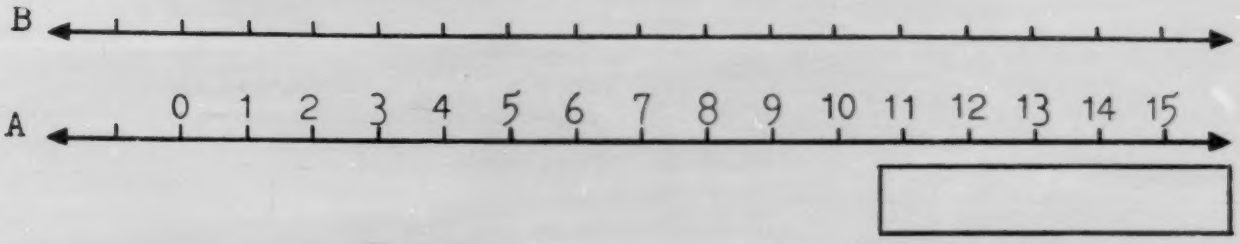
2,8



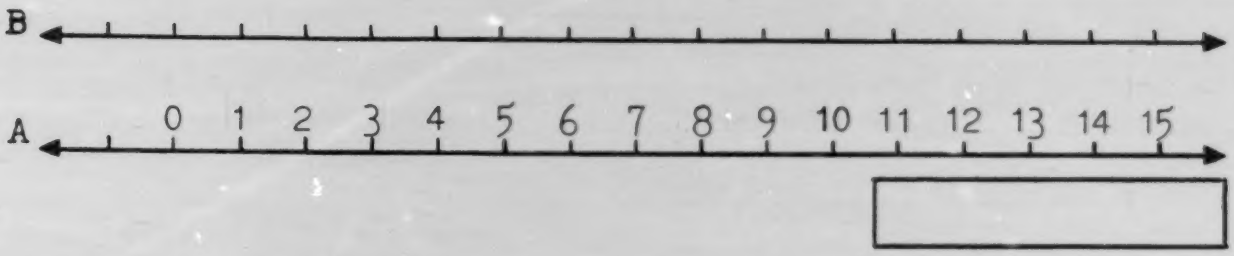
Worksheet 14 (con't)

Name_____

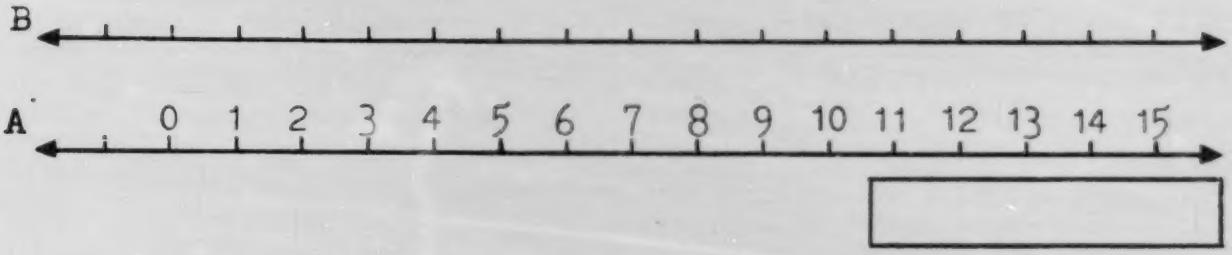
5,9



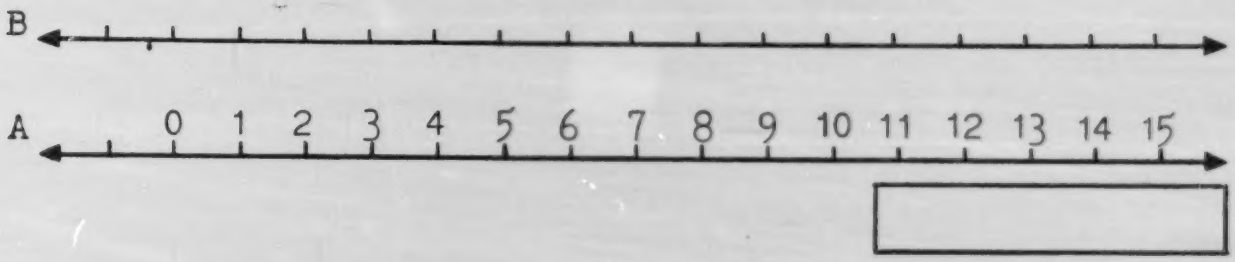
7,1



8,5



4,5

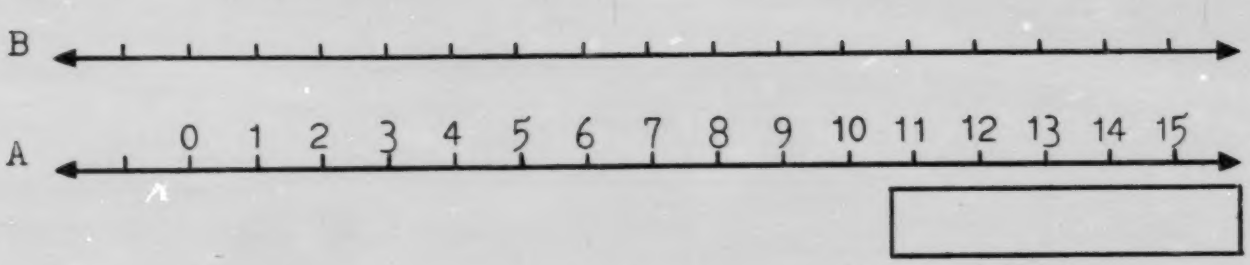


Worksheet 15

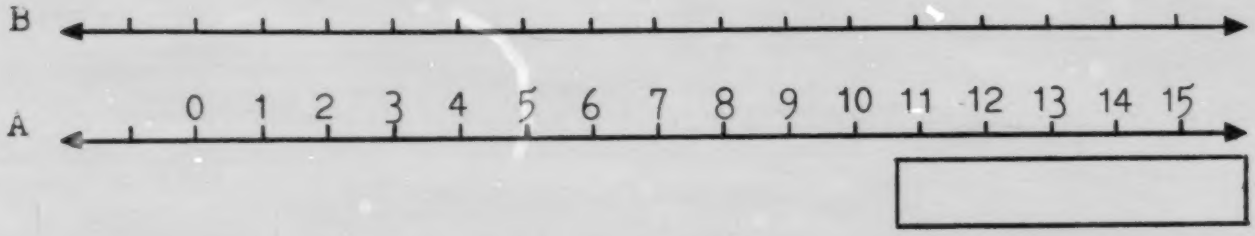
Name_____

Find the sum (con't)

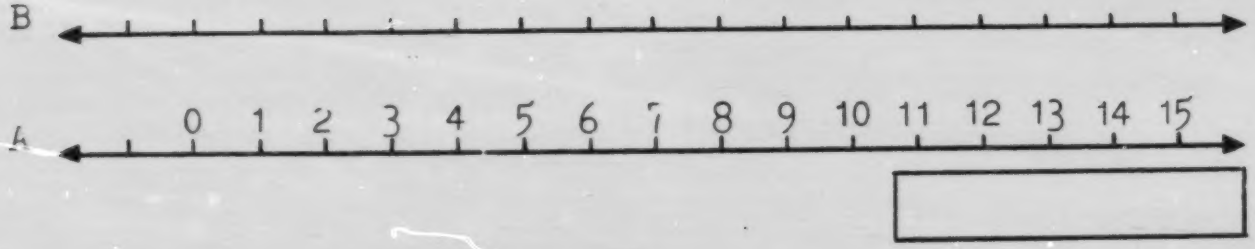
9,3



3,8



7,3



Part 2

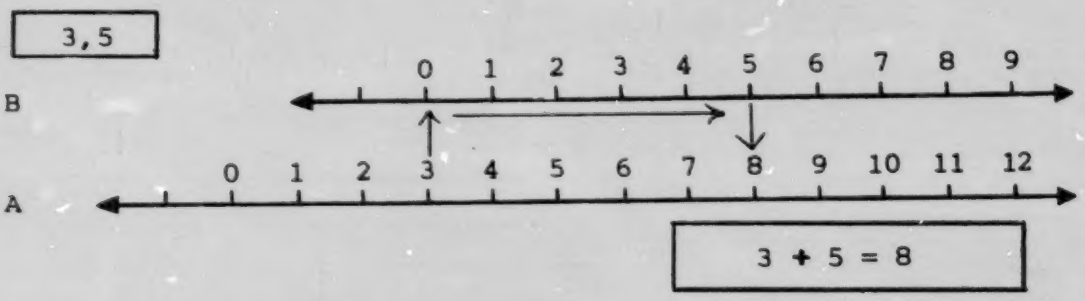
A better way

Provide each child with worksheets and Minnemast plastic scales. Insert 2 identical scales. Prepare Minnemast slide rule for a two number line frame to use in demonstrations.

Now we can use our handy plastic number lines to help us find the sums.

Who can show us the point of the first number on the classroom number frame (3).

Like this:



Yes, now we can move line B to a position where "0" is matched to that point 3.

Then we mark off the units on B to "5". It can now be matched to point "8" on line A.

Now can we understand why when you add 3 and 5 the sum is 8?

Write this number sentence in the space below the lines.
(3 + 5 = 8)

Refer to your worksheets for the number combinations.

Use your plastic two number line tool to find the sum.

Write the number sentences in the empty boxes.

Worksheet 16 . Name_____

Use your plastic scale

Find the sum.

Write the number sentence.

3,5	3 + 5 = 8	5,5	
6,3		5,7	
9,9		6,4	
9,7		6,8	
5,4		9,8	
1,9		0,4	
9,9		7,8	
2,5		7,9	
6,1		8,8	

Part 3

Whiz quiz

Provide each child with worksheets and plastic two number lines.

Attempt to develop speed and accuracy.

Place your plastic number lines on the desk near the worksheet.

When I call a number combination, find the sum on your plastic number lines.

Write the answer on your worksheet.

5,9	1,3	8,3	9,5	4,2	7,1	1,t	8,5	7,6
2,9	4,7	8,5	9,6	5,2	5,0	4,1	4,3	1,9
7,8	9,9							

After all combinations have been called the class will go over them together.

Those who complete all of them correctly may consider themselves members of the Whiz club.

Worksheet 17 Name_____

Are you a whiz?

Listen to your teacher.

Write the sum for the pair of numbers given.

a._____ h._____ o._____

b._____ i._____ p._____

c._____ j._____ q._____

d._____ k._____ r._____

e._____ l._____ s._____

f._____ m._____ t._____

g._____ n._____

Section E. Using the nomogram to find the sum

Introduction

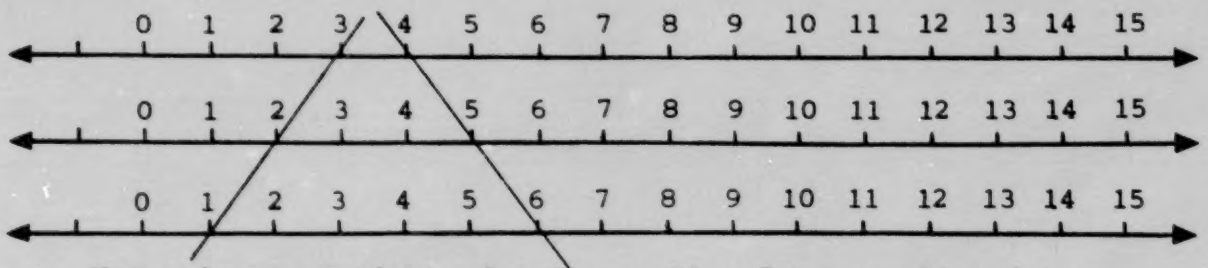
Thus far, in this unit, we have begun to see that the use of the different number lines helps us to understand the operation of addition. We are now ready to introduce a new way of using number lines and discover why it works. Before using the nomogram we will review the 2-charts in Unit XI and sums of odd and even numbers. When 2-charts were used it was found that the sum of two odd numbers or the sum of two even numbers is always even as is illustrated below:

$1 + 1 = 2$ $\qquad$ $1 + 3 = 4 = 2 + 2$

$2 + 2 = 4$ $\qquad$ $3 + 5 = 8 = 4 + 4$

$3 + 3 = 6$ $\qquad$ $1 + 5 = 6 = 3 + 3$

By considering more examples we see that any even number then can be written as a sum where both addends are the same. Let us now draw three identical number lines and line up the zeros.



Now using a straight edge draw a line from an odd number on the top line to an odd number on the bottom line or from an even number on the top line to an even number on the bottom line. If the spacing between the top and middle lines is the same as the spacing between the middle line and the bottom line, then the lines drawn as directed above will cross the middle line at the number which added to itself gives the sum for the numbers on the top and bottom lines. In our drawing

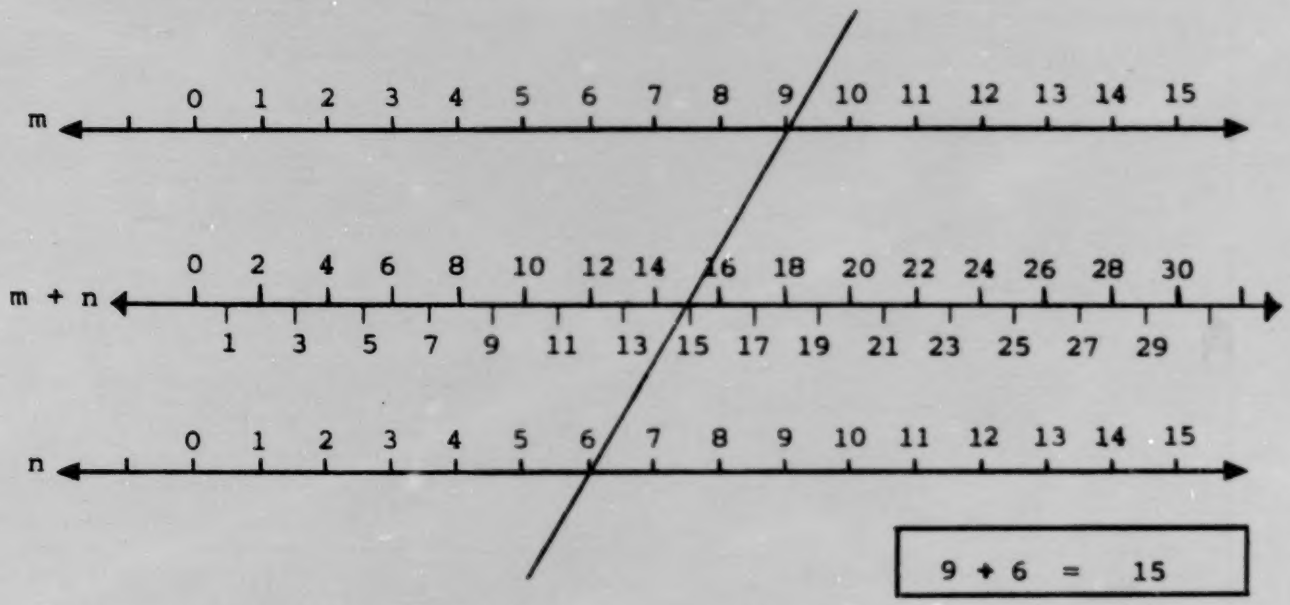
$1 + 3 = 4 = 2 + 2$ and $4 + 6 = 10 = 5 + 5$.

The children should be given many examples to discover that this is always so.

> Note: The number on the middle line is half-way between the number on the top and bottom line.

Ask them now if they see any way which the middle number line could be used to obtain the sum of a number on the top line and a number on the bottom line. If they do not discover it for themselves, suggest that the middle number line might be renumbered, 1 is replaced by 2, 2 by 4, 3 by 6 and etc. We now have our nomogram and are ready to begin using it.

Find the sum of 9,6.



a. Locate "9" on the m line and the "6" on the n line.

b. Place a ruler so that the "9" and the "6" are in a line and visible.

c. The ruler touches a point on the center line. Read this point. It is "15", the sum for 9 + 6.

Part 1

Bridge it

Draw a diagram of the nomogram on the chalkboard. Lines m and n should have 6" between the numbered points. Line m + n should have 3" between the numbered points.

Provide children with worksheets.

Today we are going to find the sum of two numbers by using the nomogram.

After carefully looking at the nomogram, who can show us how to locate the first number (9) using the chalkboard diagram?

Yes, we locate it on line "m".
(See direction sheet.)

Now, who can suggest where we might locate our second number (6) in order to find the sum.

Yes, since the points on line "n" measure off the same distances between the points as line "m" we will locate 6 on line "n".

Use your ruler to match points 9 and 6. Draw the "Bridge" which connects these points. At what point does the "Bridge" cross line m + n?

That's right. It crosses at point 15, the sum of 9 + 6.

Extend that bridge above line m so that you can write the number sentence on it.

Find the sums of the number combinations on your worksheet in the same way.

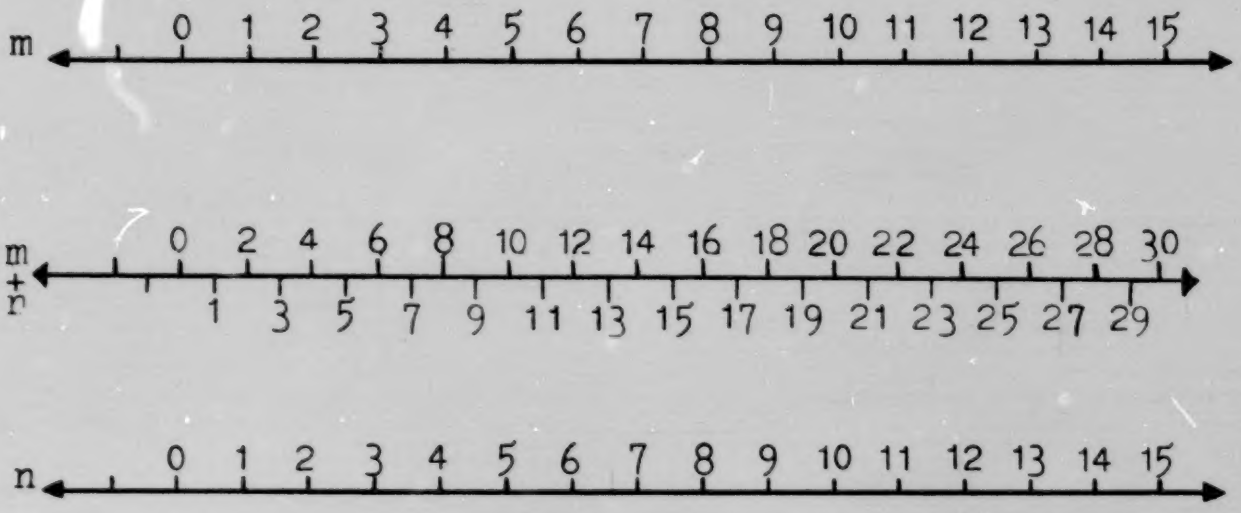
Worksheet combinations are:

13,3 12,5 11,1 15,9 5,9

Worksheet 18

Name_____

Bridge it - I

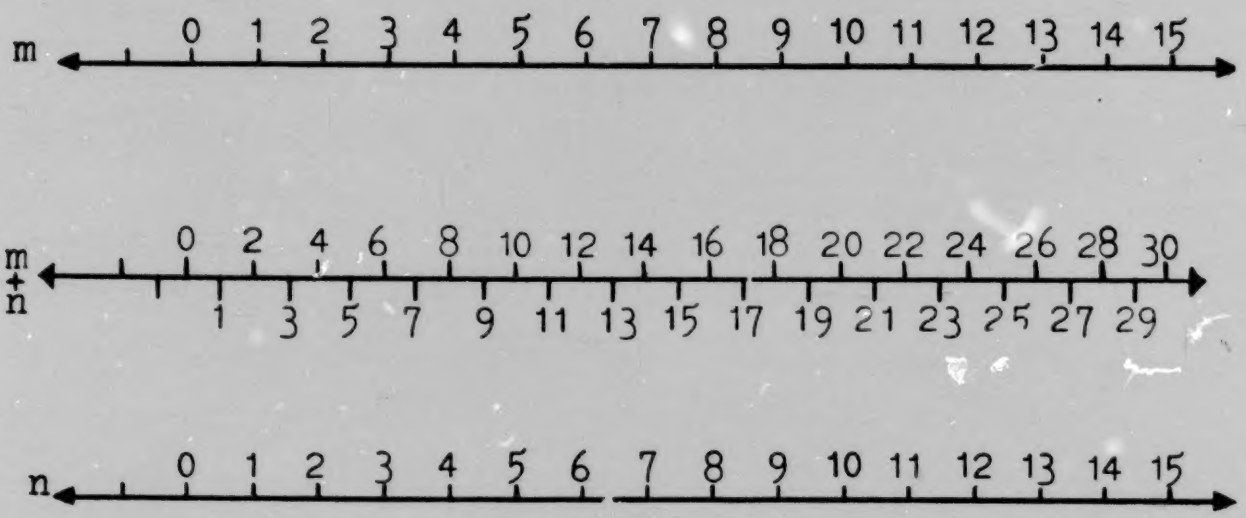


Find the sums of the following combinations by locating the points on the nomogram. Bridge the points and write the number sentence on the extended line as shown above.

13,3 12,5 11,1 15,9 5,9

Worksheet 19 Name_____

Bridge it II



Find the sums of the following combinations by locating the points on the nomogram. Bridge the points and write the number sentence on the extended line as shown above.

14,4 12,2 10,4 4,14 6,12

Part 2

Choose one

Draw a diagram of the nomogram on the chalkboard. Lines m,n should have 6" between the numbered points. Line m + n should have 3" between the numbered points.

Provide each child with a worksheet.

Let us look at group "a" on our worksheets. Here we have the combinations (9,5) (8,7) (8,8). Now, we are asked to choose the combination which we think has a sum of 15 as shown in column x.

Who would like to choose?

All right, you have chosen the combination 8,7. Let's circle it.

Now we can use the nomogram to see if our answer is correct. First, let us check each of the other combinations on our chalkboard nomogram.

Now we know that our choice was correct.

This will vary according to the children's choice.

We will now complete the worksheet in the same way.

We must prove that our choice is correct by using the nomogram.

Worksheet 20 Name_____

Finding addends

Use your plastic scale.

Circle the number pair whose sum is equal to the one given.

				Sum
a.	9,5	8,7	8,8	15
b.	17,2	16,4	4,17	20
c.	10,9	7,9	12,5	17
d.	14,9	13,9	10,11	23
e.	2,9	4,12	13,5	18
f.	11,8	13,9	9,15	24
g.	14,8	12,9	13,6	21
h.	7,15	10,11	12,15	27

Part 3

Find the hidden twins

Provide the class with worksheets and the plastic nomograms.

It is anticipated that the children will realize that different combinations can have the same sum.

Let us look at our worksheets.

In each row there are three different combinations. We are going to see if we can find the twins or those two combinations which have the same sum.

Use your nomogram to find the sums of each combination.

When you have found the "twins" write the number sentence for each in columns x and y just the way it is shown for the first group of combinations.

Complete the worksheet in the same way for each of the combinations.

Worksheet 21 Name_____

Hidden twins I

Find the hidden twins.

Circle them.

Use the nomogram to check your answer.

Write the number sentence of twins in columns x and y.

	x	y
8 + 8 6 + 9 8 + 7	6 + 9 = 15	8 + 7 = 15
11 + 7 10 + 9 9 + 9		
15 + 8 10 + 13 3 + 15		
15 + 7 12 + 9 10 + 11		
7 + 9 10 + 7 6 + 11		
12 + 13 10 + 15 5 + 19		
9 + 17. 8 + 17 11 + 14		
8 + 14 10 + 12 8 + .5		
16 + 10 15 + 12 14 + 12		

Answer Page for Worksheet 24

Hidden twins I

8 + 8	6 + 9	8 + 7	6 + 9 = 15	8 + 7 = 15
11 + 7	10 + 9	9 + 9	11 + 7 = 18	9 + 9 = 18
15 + 8	10 + 13	3 + 15	15 + 8 = 23	10 + 13 = 23
15 + 7	12 + 9	10 + 11	12 + 9 = 21	10 + 11 = 21
7 + 9	10 + 7	6 + 11	10 + 7 = 17	6 + 11 = 17
12 + 13	10 + 15	5 + 19	12 + 13 = 25	10 + 15 = 25
9 + 17	8 + 17	11 + 14	8 + 17 = 25	11 + 14 = 25
8 + 14	10 + 12	8 + 15	8 + 14 = 22	10 + 12 = 22
16 + 10	15 + 12	14 + 12	16 + 10 = 26	14 + 12 = 26

Part 4

Buddy exercise

Provide each child with plastic nomograms and worksheets.

Assign half of the class as players number I; the other half as players number II.

Place the following number combinations on the chalkboard.

9,16	8,19	17,7	18,9	14,8	20,5
15,3	19,5	14,3	16,1	18,2	17,8
13,8	20,9	11,4	10,1	11,2	19,2

Players number I are to copy in the small boxes on their worksheets the combinations as they are on the chalkboard (9,16).

Players number II are to change the order so that the numbers are reversed (commutative principle) (16,9).

Designate pairs of buddies (I and II).

Now direct them to find the sums of their combinations by using the nomogram. The number sentence should then be written in the rectangle provided.

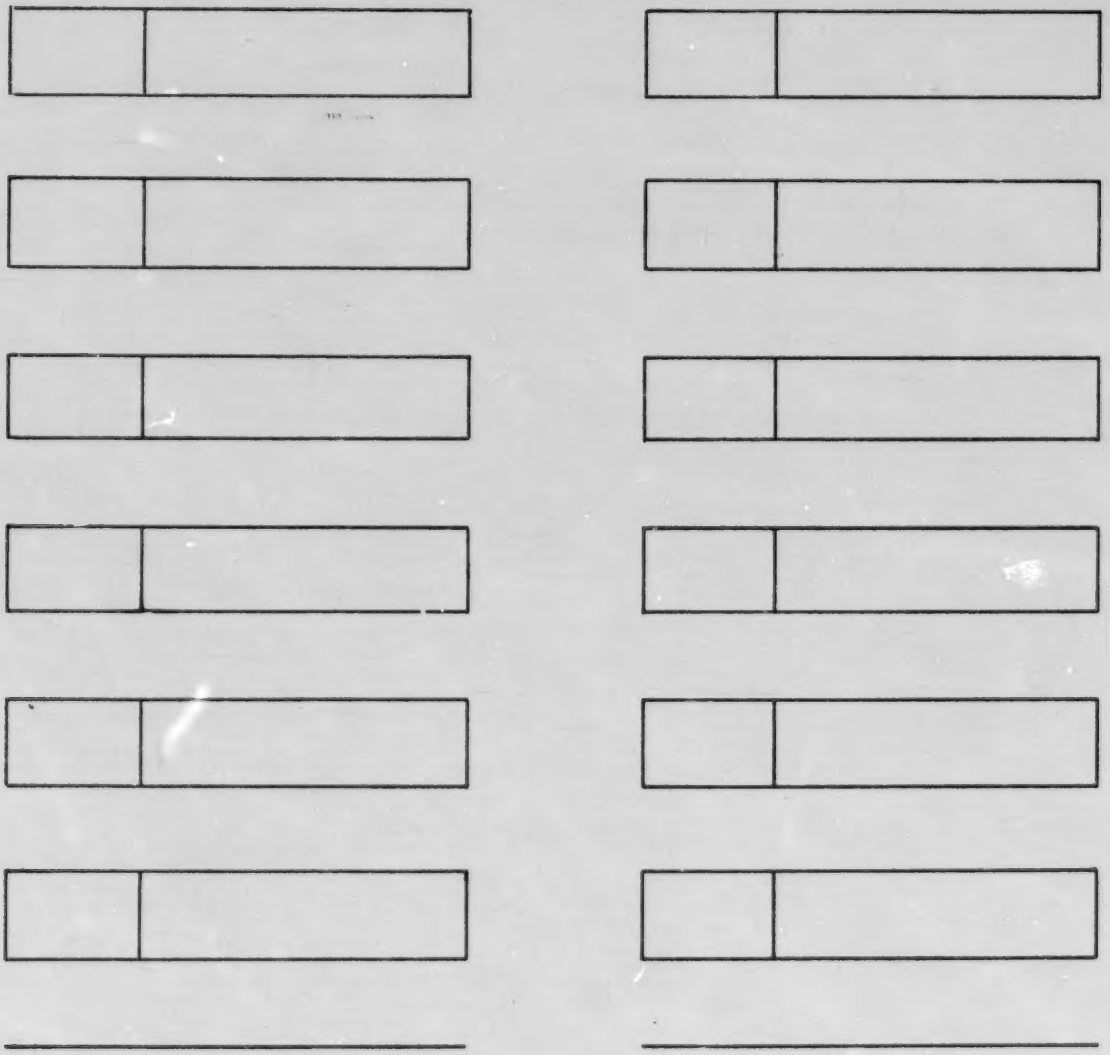
When finished, they will compare their answers.

What happens when we change the order of the numbers?

Worksheet 22 Name_____

Buddy exercise

I am player number _____.



Part 5
The "T" notation

Provide children with worksheets and plastic nomograms.

Write each step of the example on the chalkboard.

It is expected that children will reinforce their understanding of the associative law with their use of the nomogram (13 + 7 = 10 + (3 + 7) = T + T = 2T or 20).

Let us look at our first numbers 13 + 7. We are going to see if we can find the sum by grouping "tens".

We will use the nomogram to see how many tens there are in the first number 13?

"One" is correct. Write T for 10.

What can we do with the 3 ones left?

Yes, we can add it to the 7. This gives us T + (3 + 7). Show the new group in parentheses.

We will use the nomogram to add (3 + 7). The sum of 3 + 7 = 10 or T.

Now we can write it this way; T + T = 2T.

It may be necessary to go over one or two more examples with the class.

Will you now find the sums of the combinations on your worksheets. Use the nomogram to discover the different ways to group tens.

Worksheet 23

Name_____

How many T's are there?

How many T's are there?

Find the sum.

13 + 7 =	T + (3 + 7) =	T + T = 2T
16 + 4 =		
22 + 8 =		
18 + 12 =		
9 + 11 =		
14 + 16 =		
8 + 12 =		
15 + 5 =		
17 + 13 =		
18 + 12 =		

Answer Page for Worksheet 26

Discovering tens

13 + 7 = T = (3 + 7) = T + T = 2T

16 + 4 = T + (6 + 4) = T + T = 2T

22 + 8 = 2T + (2 + 8) = 2T + T = 3T

18 + 12 = T + T + (8 + 2) = 2T + T = 3T

9 + 11 = (9 + 1) + T = T + T = 2T

14 + 16 = T + T + (4 + 6) = T + T + T = 3T

8 + 12 = (8 + 2) + T = T + T = 2T

15 + 5 = T + (5 + 5) = T + T = 2T

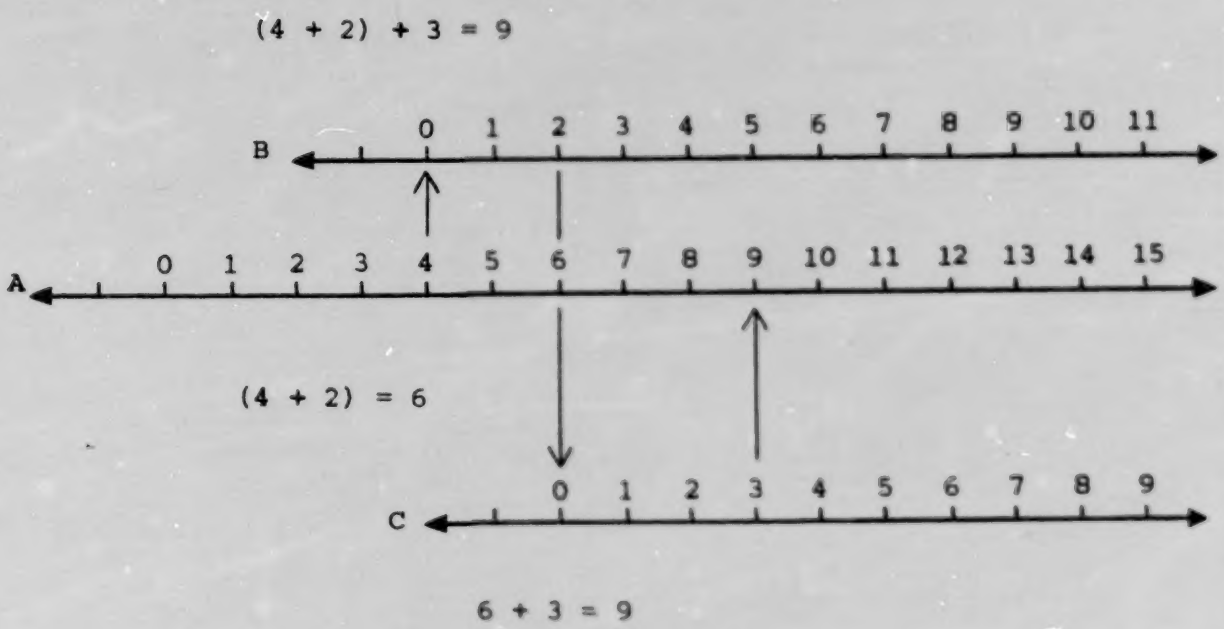
17 + 13 = T + (7 + 3) + T = T + T + T = 3T

18 + 12 = T + (8 + 2) + T = T + T + T = 3T

Section F. Using the associative property

Before proceeding, the teacher should reread the background material on associativity and commutativity. Help the children to first understand that addition can only be performed on two numbers at a time. Then lead them to discover how it is possible to talk about the sum of three or more numbers.

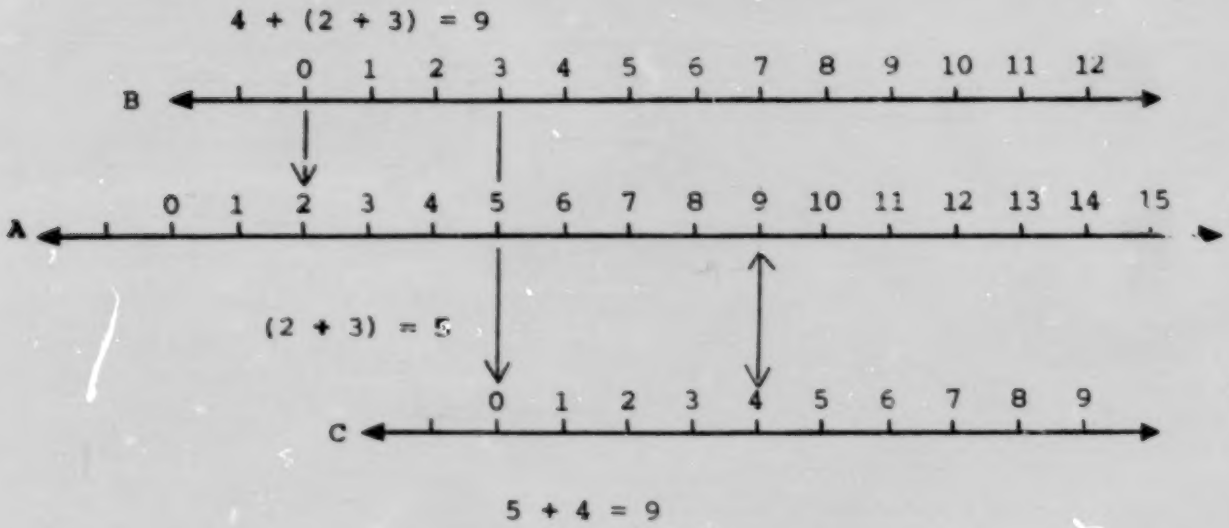
The Minnemast slide rule can be used to illustrate the associative property, use three identical scales.



To illustrate the associative property on the number scales as shown above note that scales are lettered A, B and C.

On number scale A, locate point "4". Align "0" of scale B with that point. Locate addend "2" on scale B. Arrow now points to sum of (4 + 2) or "6" on scale A. Align "0" of scale C with point "6" of scale A. Locate addend "3" on scale C. Arrow now points to scale A and shows sum of 6 + 3 or 9.

or



To illustrate that the associative property is unaffected by the order of grouping we again use number scales lettered A, B, and C.

On number scale A locate point "2". Align "0" of scale B with tnat point. Locate addend "3" on scale B. Arrow now points to sum of (2 + 3) or 5 on scale A. Align "0" of scale C with point "5" of scale A. Locate addend "4" on scale C. Arrow now points to scale A and shows sum of 4 + 5 or 9.

Worksheet 24 Name_____

Grouping I

Group any two addends.

Then find the sum.

Use your plastic number scales.

3 + 5 + 2 = _____	7 + 2 + 3 = _____
1 + 7 + 4 = _____	8 + 2 + 1 = _____
5 + 1 + 3 = _____	8 + 4 + 2 = _____
3 + 4 + 2 = _____	6 + 2 + 3 = _____
2 + 1 + 6 = _____	5 + 3 + 4 = _____
4 + 3 + 1 = _____	6 + 1 + 3 = _____
5 + 2 + 3 = _____	4 + 5 + 4 = _____

Worksheet 25 Name_____

Grouping II

Group any two addends.

Then find the sum.

Use your plastic number scales.

6 + 5 + 4 = _____	8 + 7 + 6 = _____
5 + 9 + 5 = _____	3 + 9 + 7 = _____
4 + 7 + 8 = _____	6 + 9 + 8 = _____
7 + 3 + 8 = _____	9 + 8 + 7 = _____
9 + 4 + 8 = _____	7 + 9 + 6 = _____
9 + 7 + 9 = _____	5 + 8 + 8 = _____
8 + 5 + 9 = _____	4 + 9 + 7 = _____

Worksheet 26

Name_____

Review

$2 + _____ = 5$

$5 + 4 = _____$

$_____ + 4 = 7$

$2 + 7 = _____$

$(2 + 5) + 4 = _____$

$5 + _____ = _____$

$2 + (4 + 4) = _____$

$8 + (3 + 3) = _____$

$16 + 7 = _____$

$_____ + 23 = 31$

$_____ + 18 = 26$

$29 + _____ = 34$

$(6 + 4) + 5 = _____$

$(7 + 3) + _____ = 20$

$8 + _____ = 17$

$_____ + 11 = 23$

$_____ + (4 + 9) = 15$

$(23 + 2) + _____ = 30$

$(8 + _____) + 10 = 20$

$7 + (7 + 8) = _____$

Section G. Using the nomogram to find the missing addend

Materials

1. Minnemast slide rule
2. Plastic nomograms
3. Transparent straight edges (Pull scale from slide rule.)

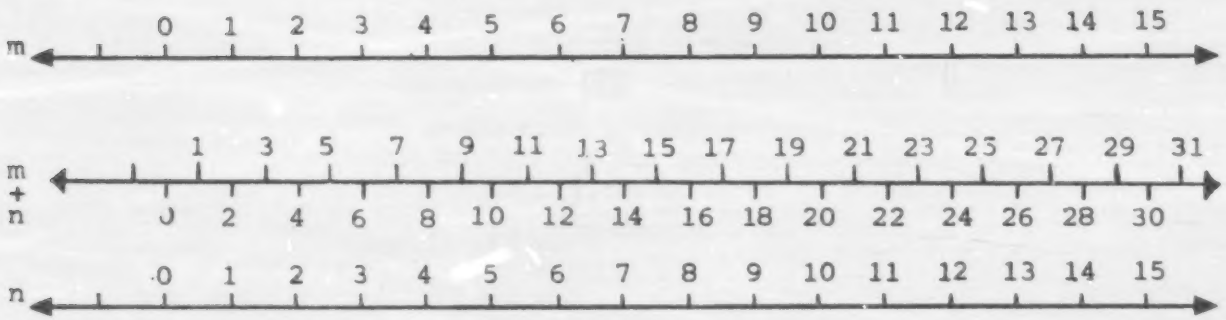
Study the diagram below until you can demonstrate with the slide rule.

Then permit the children to use their plastic nomograms and straight edges.

Follow with worksheets. Use the number pairs found on random card C.

To find a missing addend on a nomogram.

$9 - 6 = 3$



a. Begin with the center line marked m + n. Locate "9".
b. Locate "6" on the m line.
c. Place the straight edge so it is in line with "9" and "6".
d. Look at line n. Read the missing addend. It is "3".

Worksheet 27 Name_____

Random card C

Row							
1	10,1	16,6	18,5	11,2	20,9	13,3	19,8
2	12,3	19,7	14,3	17,5	11,5	13,2	18,3
3	16,2	11,3	17,9	16,4	15,3	10,2	19,6
4	1,17	12,8	18,7	11,6	12,4	14,2	15,8
5	11,4	13,4	19,5	10,8	16,5	14,1	17,1
6	14,7	18,8	10,3	15,7	11,1	16,4	19,4
7	16,8	17,7	13,9	20,7	10,9	18,1	19,3
8	15,5	12,6	14,9	17,6	13,1	18,9	10,6
9	13,7	10,5	14,6	19,2	11,8	16,3	12,5

Worksheet 28 Name_____

Make your choice

Choose 3 pairs of numerals from each row. Use your plastic nonogram to help you find the missing addend.

Then write a number sentence.

1			
2			
3			
4			
5			
6			
7			
8			
9			

Worksheet 29 Name_____

A race for two

Who is your partner?

Use a plastic nomogram and card C.

Race to find the missing addends for the first pair of numbers in these rows.

Write the missing addend:

Row	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Worksheet 30

Name_____

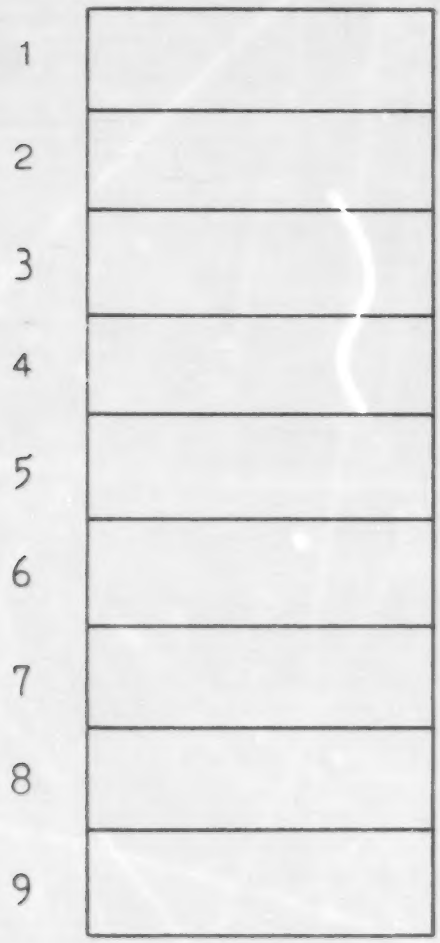
A race for three

Arrange a group of 3 children.

Use your plastic nomogram and random number card c.

Write only the missing addend for the last pair of numbers in each row.

Row



The winner is_____.

Section H. Making and reading an addition chart

Directions for making an addition chart

Have the pupils add to the zero in the left column each of the numbers across the top row.

Write the sum under the number which was added to zero.

Continue with the rest by adding the column number to the row number. (Note the pupil's worksheet.)

It is a good idea to make a large room chart with the class.

Have the children practice locating the addends and the sum.

Column

Row	+	0	1	2	3	4	5	6	7	8	9
	0	0	1	2	3	4	5	6	7	8	9
	1	1	2	3	4	5	6	7	8	9	10
	2	2	3	4	5	6	7	8	9	10	11
	3	3	4	5	6	7	8	9	10	11	12
	4	4	5	6	7	8	9	10	11	12	13
	5	5	6	7	8	9	10	11	12	13	14
	6	6	7	8	9	10	11	12	13	14	15
	7	7	8	9	10	11	12	13	14	15	16
	8	8	9	10	11	12	13	14	15	16	17
	9	9	10	11	12	13	14	15	16	17	18

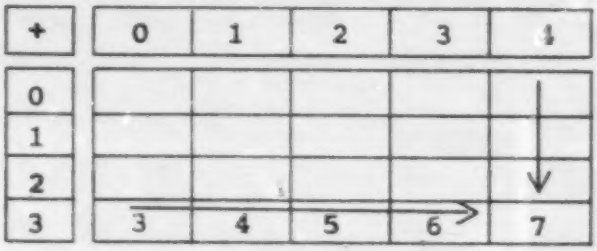
Suggestions for practice on the room addition chart

1. To find a sum when two addends are known:

   a. Teacher (or pupil) writes a number sentence on chalkboard.

      For example: 3 + 4 = X

   b. Another pupil finds "row 3" and moves <u>to the right</u> along this row until he is pointing to the numeral directly under the column heading 4. This will be "7".
   Like this:



   c. Pupil writes the number sentence giving the sum: 3 + 4 = 7.

2. To find a missing addend when one addend and the sum are known:

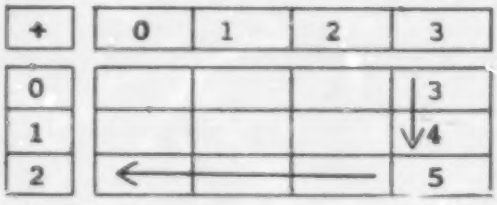
   a. Teacher (or pupil) writes a number sentence on the chalkboard.

      For example: x + 3 = 5

   b. Another pupil finds "columns 3" and moves the pointer <u>down</u> to the numeral "5".

   c. From the numeral "5" he moves <u>to the left</u> to locate the row number which is "2".

      Like this:



      The missing addend is "2".

   d. The pupil writes the completed number sentence: 2 + 3 = 5.

Note: Whenever possible, have the children discover this relationship for themselves.

Worksheet 31

Name_____

An addition chart

Column ↓

Row →

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

Worksheet 32 Name_____

Read your chart

Use the chart to find these sums.

How quickly can you work?

8 + 9 = _____	6 + 4 = _____
9 + 5 = _____	8 + 5 = _____
4 + 8 = _____	9 + 3 = _____
7 + 6 = _____	9 + 9 = _____
8 + 8 = _____	6 + 6 = _____
9 + 7 = _____	7 + 8 = _____

Worksheet 33 Name_____

Read your chart

Read the chart to find the missing addend.

How can you check your work?

12 - 4 = _____	14 - 6 = _____
13 - 9 = _____	12 - 5 = _____
16 - 8 = _____	13 - 7 = _____
11 - 5 = _____	15 - 8 = _____
17 - 8 = _____	11 - 9 = _____
11 - 4 = _____	14 - 7 = _____

Section I. Using letters for the unknown number

The following exercise introduces the use of a letter (in this instance "x") to represent the unknown number.

It is important that the children understand that any letter may be used for this purpose. If we are trying to find the cost of an item, the unknown may be designated by the letter "c". If our problem is to find the number of people who visited the World's Fair on a given day, we may choose to use the letter "n" or "p". Similarly, in the "Let's play detective" the letter "x" is appropriate to indicate the unknown number.

Make your introduction brief and light. Do not dwell on it.

Worksheet 34 Name_____

Let's play detective I

I am thinking of a number.

I won't tell you what it is, but I will call it "x" for the unknown. I will give you a clue. If you add 2 to the "x" you get 5. What is the number?

$2 + x = 5$ $x =$ ___

$2 + x = 5$ $x = 3$

How many can you solve?

$3 + x = 7$ $x =$ ___	$x + 4 = 9$ $x =$ ___	$4 + x = 8$ $x =$ ___
$x + 5 = 11$ $x =$ ___	$4 + x = 6$ $x =$ ___	$2 + x = 10$ $x =$ ___
$6 + x = 9$ $x =$ ___	$x + 5 = 10$ $x =$ ___	$x + 6 = 12$ $x =$ ___
$8 + x = 11$ $x =$ ___	$x + 9 = 11$ $x =$ ___	$x + 7 = 14$ $x =$ ___

Worksheet 35 Name_____

Let's play detective II

x + 4 = 6 x = ___	x + 4 = 6 x = 2

How many can you solve?

x + 5 = 8 x = ___	x + 5 = 7 x = ___	4 + x = 10 x = ___
x + 8 = 12 x = ___	3 + x = 12 x = ___	2 + x = 7 x = ___
2 + x = 6 x = ___	x + 4 = 11 x = ___	x + 8 = 15 x = ___
x + 5 = 13 x = ___	x + 6 = 11 x = ___	6 + x = 15 x = ___
7 + x = 14 x = ___	x + 10 = 20 x = ___	5 + x = 12 x = ___
9 + x = 13 x = ___	15 + x = 21 x = ___	x + 8 = 16 x = ___

Section J. Finding answers to problems

Work the problems on the first three pages with the class.

Have them read the problem and consider the questions on the following pages.

Have the child write a number sentence for the problem.

For example:

Problem 1: $4 + 9 = x$

Problem 2: $6 + 3 = x$

In order to find a sum the addends are combined.

If an addend is missing the sentence would be rephrased as:

$3 + x = 9$.

Use the problem pages as you like to suit the needs of your class.

Story Problems

Unit XII

STORY PROBLEMS

Ask yourself these questions as you work:

A. What do I want to find out?
   1) Underline the key words that help me decide.

B. Look at each number which could help me.
   1) Is it an addend?
   2) Is it a sum?

C. What kind of a number do I need?
   1) Is it a missing addend?
   2) Is it a sum?

D. What will my number sentence be?

E. How can I find the answer?
   Can any of these ways help me?
   1) Drawing a picture.
   2) Using a number line.
   3) Using the nomogram.
   4) Reading the addition or subtraction chart.
   5) Best of all, do I know the number fact?

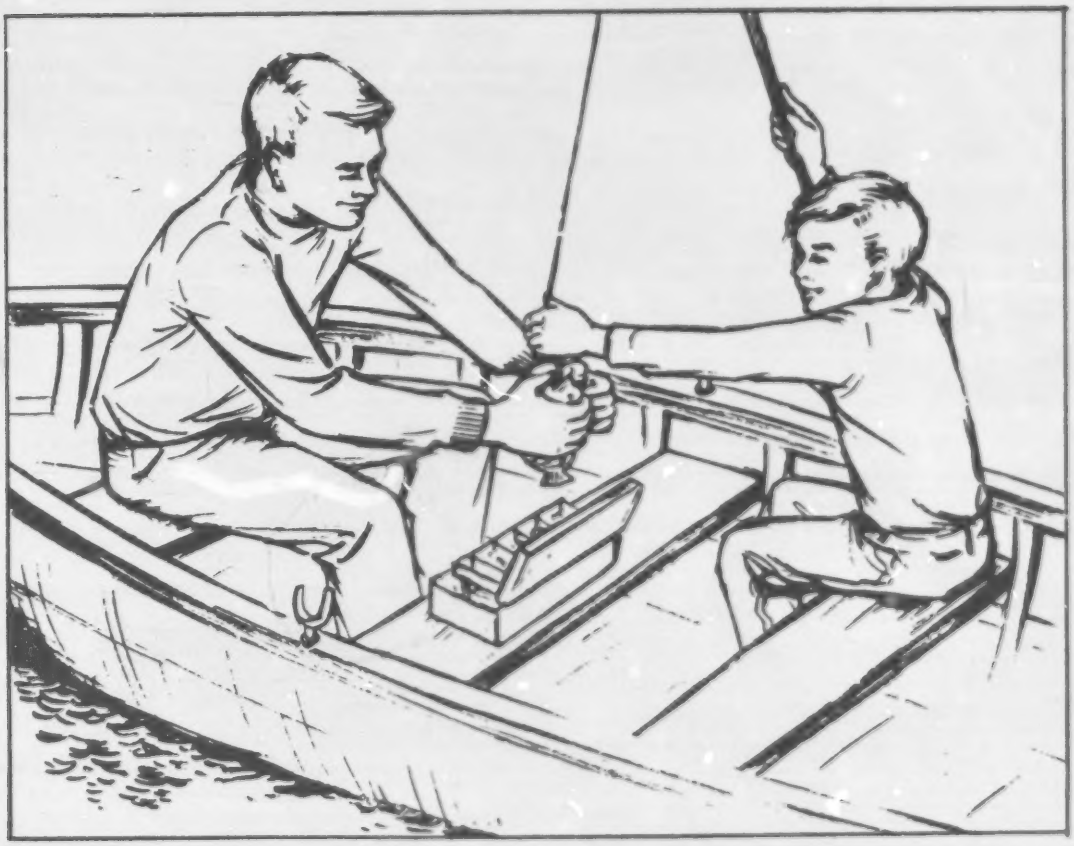
STORY PROBLEMS

1) Fishing with Dad was great fun! That is what Jack thought one day as he pulled up fish after fish. He soon had 4 big sunfish.

Dad was doing even better! He caught 9.

"What a catch we have!" cried Jack. "Let's see, how many fish do we have here?"

Can you find the answer to Jack's question?



STORY PROBLEMS

2) The second Grade Kitten Show was held on Monday. Some children put pretty colored bows on their kittens. Some brought balls for their pets.

How excited everyone was to see and hear about each kitten!

Six girls and three boys brought kittens to school that day.

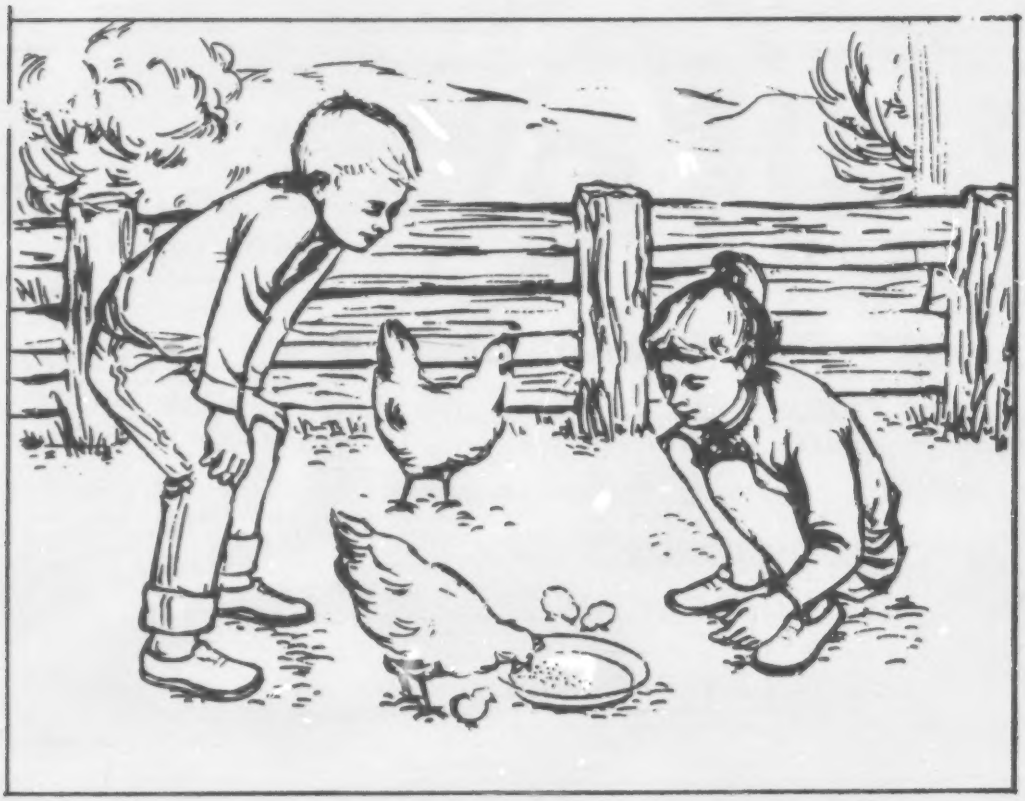
How many kittens were in the show?

3) How happy Jane and Jerry were to spend last summer with Grandmother and Grandfather on the farm!

They helped with the work. Jane took care of 5 chickens. Jerry had 11 chickens to feed.

Jane said, "Jerry, chickens always seem to be hungry. You have more to feed than I do. How many more, Jerry?"

What was Jerry's answer?



STORY PROBLEMS

4) Jill was a busy girl. She was picking apples from the old apple tree. She picked 10 big red apples. "Jill," called Mother. "May I have 6 of those to make a pie for dinner?"

"Oh, yes!" cried Jill. "And I am taking the rest to Mr. and Mrs. Smith."

How many apples did Jill take to the Smith house?

5) Judy was having a birthday party. Mother brought 12 white and 6 pink cupcakes outside for the children.

"Oh, dear!" she said. "Judy, how many more white cupcakes do we have than pink ones?"

Can you help Judy answer her mother?

6) John and Bob had a picnic whenever they could. They invited some other boys to go with them. You are right if you think they took hot dogs to roast. John brought 12 and Bob 6 more.

How many hot dogs did the boys have at the picnic?

7) February 14 is Valentine's Day. Alice had 9 very very good friends. She wanted to make each one a lovely valentine.

One afternoon she made 6 of them.

"Alice," Mother asked. "How many valentines do you have left to make? It is almost time for dinner."

What was Alice's answer?

8) Johnny lived a long, long way from school. After he walked 4 blocks, he had 5 more to go.

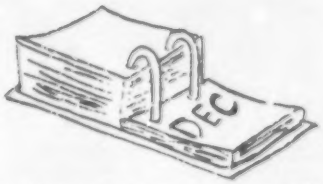
How many blocks did Johnny live from school?

STORY PROBLEMS

9) "Will Christmas never come!" cried Pat. "It is 14 days till then."

"We will shop the first 5 days," said Dad.

"Then how many days will I still have to wait?" asked Pat.

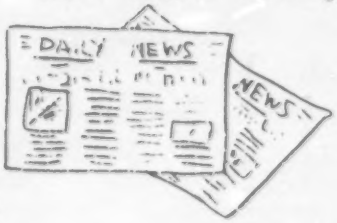


Can you answer Pat's question?

10) Jack liked to run errands to the drugstore. There were so many things to look at there!

So when Mother asked, "Jack, will you buy an evening paper for Mrs. Clark and another one for Dad," he was eager to go.

The papers cost 7¢ apiece. How much money did Jack need?



11) Do you like to read books?

Mary does! She read 7 books in May. In June, because it was vacation time, she read 12.

Can you tell how many more she read in June than in May?

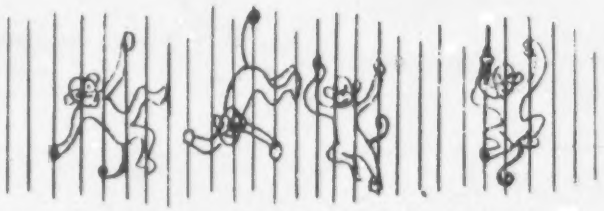
STORY PROBLEMS

12) Bill was saving his money. One day he had 18¢.
Then Uncle Ted gave him 5¢ more.
Bill sat down to count his money.

How much did he have?

13) Going to the zoo is such great fun! Jeff and Dave like to feed the monkeys.
Jeff gave the monkeys 8 peanuts. Dave fed them 15.

How many more peanuts did Dave feed the monkeys?



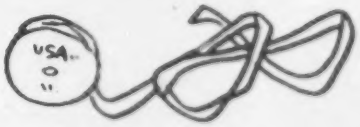
14) Baby Sue was playing with her blocks. She had 12 blue ones and 7 red ones.

Baby Sue couldn't count, but can you tell how many blocks she had?

15) Peggy and Alice loved to wear ribbon bows in their hair.

Peggy had 16 inches of white ribbon. She cut off 8 inches for Alice.

How many inches did she have left for her bow?



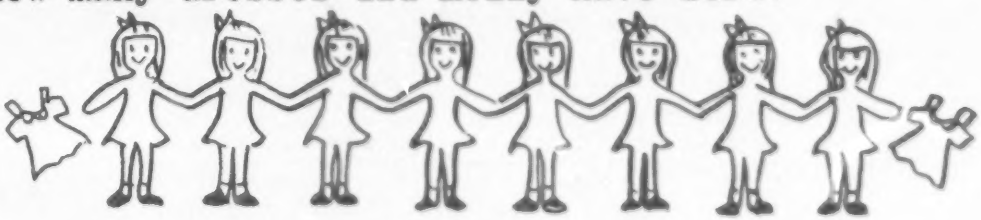
16) Rabbits! Rabbits! Rabbits! Tom had 17. Bob had 8.

How many more did Tom have than Bob?

17) One afternoon Molly was playing with her paper doll. The doll had 12 dresses.

While Molly played, she lost 5 of them.

How many dresses did Molly have left?



STORY PROBLEMS

18) Dad gave Bill his allowance.
Bill decided to spend it all. He bought a ball for 15¢ and a candy bar for 10¢.

What was his allowance?

19) Mark collected toy airplanes. He had 11.
Last year he got 9 more as gifts.

How many does he have now?

20) Linda gave her friends some candy she made. She put 9 pieces on a beautiful blue plate.
Her mother put 4 more pieces on a pink plate.

How many pieces of candy were there in all?

STORY PROBLEMS

21) Bob and Jack played together every day. One day Bob took out 7 toy cars.
Bill brought out 3 more.

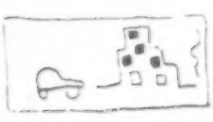
How many toy cars did the boys have to play with that day?

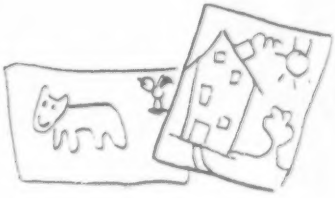
22) "Robins are on the lawn," said Dick. "I can see 8 of them."
"There are 4 more over there," said Sam. "They are all looking for worms."

How many robins were on the lawn?

23) Do you like to draw pictures? Jane and Mary did. They used each others' crayons to color their pictures.
Jane had 9 crayons and Mary had 6.

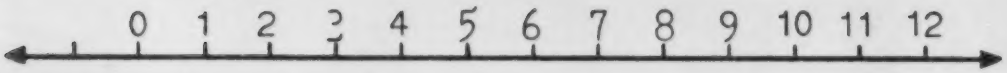
How many crayons did they have to use together?

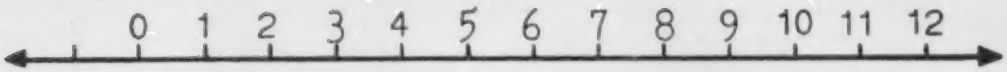


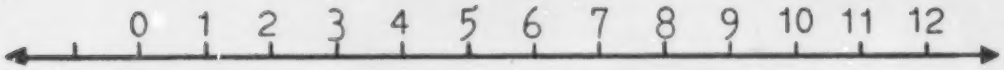


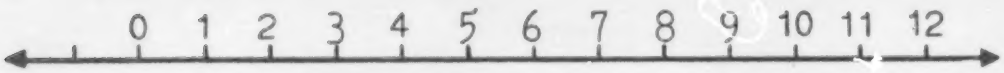
STORY PROBLEMS

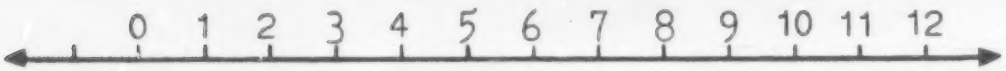
Number lines to use for story problems

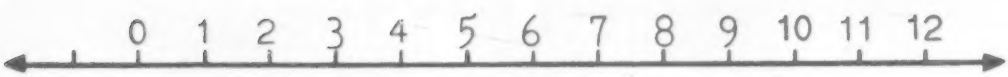












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